The Limits of Power Control in Wireless Networks: The Two-Sender Case

ABSTRACT

This paper analyzes the effect that sophisticated power control mechanism might have on total throughput of two-sender wireless networks. The paper's main conclusion, based on theoretical and experimental studies, is that for any two sender topology, the best strategy is either for the stations to send one at a time, or for them to send concurrently at the maximum power level of which they are capable and that there is no use of any adaptive power control mechanisms.

A simple experimental analysis in the paper suggests that competing 802.11 signals looked like white noise and thus interference can be assumed to appear as white noise to the receivers. The paper's theoretical analysis uses Shannon's capacity formula to estimate how much simultaneous senders will decrease each others' throughput due to interference. The analysis assumes that each sender chooses a transmit bit-rate that will maximize throughput to its receiver given the current attenuation and interference. The analysis derives the general expression for the transmit power that maximizes the throughput for the two-sender physical topologies. This expression is found to correspond to the cases where one node transmits or both the nodes transmit at a time with maximum transmission power.

The paper goes on to present measurements from an 802.11 test-bed for the two-sender topology, in order to see if the conclusions from the theoretical analysis hold in practice. The measurements search for the highest-throughput combination of transmit power levels is with carrier sense and other deferral mechanisms off. The test-bed measurements also support the conclusion that adaptive power control is not beneficial.

1. INTRODUCTION

Throughput in wireless data networks is often disappointingly low, so techniques to increase throughput are welcome. One of the most powerful throughput-increasing techniques is spatial re-use: allowing concurrent transmission by nodes that are far enough apart that they don't significantly interfere. Spatial re-use often occurs naturally, in cases where nodes are not aware of each other. But denser networks must take explicit steps to exploit spatial re-use when possible. That is, when more than one node has data to send, the network's medium access control (MAC) protocol must decide whether total throughput will be maximized by concurrent or one-at-a-time transmissions and what transmit power levels to be used by the nodes.

Simplifying somewhat, the throughput at each receiver is determined by the signal-to-interference-noise ratio (SINR) at the receiver, which in turn is determined by the transmit power levels of all concurrent senders and the attenuation from the senders to the receiver. The node sending to its receiver (assuming there is only one such sender) contributes the signal, and the other senders' transmissions contribute to noise at that receiver. The MAC cannot change the attenuation, but it can potentially choose the subset of nodes that send at any given time, the transmit power levels they will use, and the bit-rates at which they will transmit. These are the main tools with which the MAC can attempt to maximize total throughput. MAC protocols have traditionally focused on choosing which nodes should send and when. Modern wireless MAC protocols (such as those in commodity 802.11 radios) also incorporate adaptive bit-rate selection, in order to send with the bit-rate which will result in the highest throughput for the current SNR at the receiver. Choice of transmit power level, however, is still an active research area.

This paper explores the effects that power control might have on total throughput for the case of two senders (more would be nice, but even two results in significant complexity in the analysis). It makes assumptions that are suitable for modern 802.11 radios, in particular adaptive bit-rate selection. It assumes that the goal is a new MAC that will not be bound by existing techniques such as carrier sense and 802.11's RTS/CTS. The paper does not propose a new power-control MAC, but evaluates how useful such a MAC might be.

The paper uses two tools to analyze power control. First, it presents theoretical derivations that use Shannon's capacity to predict the effects of transmit power levels on various receivers' signal and noise levels and thus on how much throughput they can expect. The paper illustrates the theoretical results with simulations of various network topologies and explanations of the corresponding best power-control strategies. Second, the paper presents a set of measurements taken from an 802.11 test-bed. These measurements are necessarily more restricted in scope than the analysis and simulations, but they do serve within their limits to validate the paper's conclusions.

The paper's main conclusion is that virtually in every two-sender configuration, one of two strategies results in highest total throughput. Either one node should send at a time at the highest possible power level (while the other is silent), or both nodes should send at the highest possible power level. It is never the case that a power level other than the maximum is required in order to maximize throughput. Thus a MAC needs to only decide whether a node should send and what bit-rate it should use; it need not separately choose a power level. This does not imply that a MAC such as 802.11's is likely to maximize total throughput, since a correct decision about whether to send requires knowledge of what transmissions are already underway and what SINR values the receivers of such transmissions are experiencing. But it does imply that a sophisticated MAC need not include a power control mechanism.

The rest of the paper proceeds as follows. The next section describes existing work in the area of power control. Section 3 explains the underlying assumptions made throughout the paper regarding radios, packet reception and channel characteristics. The theoretical portion of the paper in Section 4 derives the expression for optimal power combination and illustrates them through examples. The experimental results in Section 5 verify the theoretical claims for 2-sender case on a simple 2-sender test-bed. It would be interesting to analyze the benefits of power control for n-sender networks. Based on the analysis of 2-sender networks, it is possible to intuitively understand to some extent what happens in the n-sender networks. Section 7 introduces the effects of power control in n-sender case and lists out the possible future work in this direction and finally concludes the paper.

2. RELATED WORK

A major fact exploited in this paper is that the interfering signals appear as white noise to the receiver. This does not imply that a receiver can always decode even the weakest signal in the presence many stronger signals. The ability of the receiver to decode a signal, depends on the ratio of the strength of its signal (due to the transmission from its intended sender) to the sum of its interference (due to the transmissions from other neighboring senders) and background noise, which is nothing but the SINR at the receiver. Section 3 discusses interference from neighboring senders, in detail and provides an experimental verification.

Though many existing papers [x,y,z] have used the concept of SINR, most of these existing work on transmit power control relies on an abstraction of radio behavior that is often called the Fixed Transmission Range (FTR) model. The FTR model predicts that a node's transmissions will be heard only by receivers within a range that is determined by the transmitter's power level; similarly, a transmission will only cause interference at receivers within that range. One implication of the FTR model is that two sender/receiver pairs that are within range of each other cannot productively send at the same time: the transmissions will interfere and the receivers will receive no useful information. A power control protocol can exploit the FTR model by observing that, if the senders reduce their power levels enough, their radio ranges will no longer overlap, and they can then send concurrently without interference; the FTR model predicts that the result would be a doubling of total throughput. The subsequent discussions in Sections 3 and 4 would prove that the FTR model's implication that decreasing power will increase total throughput is not correct for these radios based on the assumptions used in this paper.

One example of work that assumes the FTR model is [1, 2, 6], which suggests that power control algorithms could reduce the number of interfering access points. Similarly, [3, 8] predicts that a low common power will produce a disconnected network while high power will produce interference.

There are some limited senses in which the FTR model is correct. If nodes do not have adaptive bit-rate selection, then low SNR values or long distances may prevent communication (in situations where a more sophisticated radio would reduce its bit-rate), and high SNR values are wasted (do not result in higher bit-rate and thus throughput). If nodes are required to use carrier sense, then at most one of a set of nearby nodes will transmit, where "nearby" depends on the carrier sense threshold; carrier sense suppresses transmission whether or not the transmission would actually result in packet loss or decreased throughput. While these effects are of great importance with some radio hardware, they are not fundamental, and can be expected to decrease in significance as radio hardware becomes more mature. For example, modern 802.11 radios can adapt their bit-rates over multiple orders of magnitude, and can also operate without carrier sense if higher layers are able to make better decisions about when to send. With such radios and AWGN channel, decisions about whether to send and at what power level should be made with an entirely different analysis that uses SINR ratios to predict throughput, as in Section 4.

The authors of [4] and [7] make similar observations, predicting that a universally scaled power level for all the nodes in the network does not affect the total throughput of the network. Shepard [7] uses a sophisticated SNR analysis, but intentionally fixes the bit-rate (for simplicity) and carefully engineers the rest of the system to produce predictable SNR ratios. However, our paper goes a step ahead and claims that there is no use of sophisticated power mechanisms for two-sender networks.

An area of investigation related to power control analyzes the situations in which concurrent transmission increases total throughput [3, 5, 8]. A frequent conclusion is that even for two-sender topologies, there is more potential for spatial reuse using concurrent transmission with adaptive transmit power control mechanisms than carrier sense allows. Though these papers conclude that there is some use of concurrent transmissions, they still believe in the FTR model to calculate the power level which the nodes should use for transmission.

3. ASSUMPTIONS

The theoretical analysis in this paper makes a number of simplifying assumptions about the behavior of 802.11 radios, in order to keep the analysis focused on fundamental behavior. This section explains the assumptions made about the radio model, the wireless channel, and how packets are received. Much of the underlying behavior described in these models is also verified experimentally.

3.1 Interference from other senders

In an effort to mimic the behavior of 802.11 radios, the radios only receive a single transmission at a time and have a minimum decoding threshold of 6dB above the noise floor.

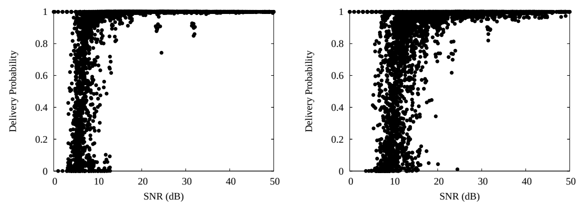

Figure 1: Delivery probability for single senders versus SNR for 6 and 12 Mbps.

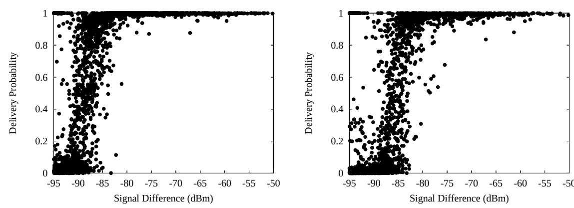

Figure 2: Delivery probability versus signal difference when two nodes are sending. The signal difference required for delivery at 6 and 12 Mbps are similar to those in Figure 15, suggesting that other 802.11 signals do not destructively interfere.

The threshold is the minimum signal to noise ratio required for a typical 802.11a radio to decode a frame at the slowest rate of 6 Mbit/s.

As a result of the single packet reception assumption, when multiple signals arrive at a receiver, at most the strongest signal may be decoded if it is 6dB stronger than the combined interference from all the other transmissions. The interfering signals are assumed to appear as white noise to the receiver and add together linearly. Thus SINR (signal to interference and noise ratio) is used to describe the throughput of a network in the presence of interference from one or many neighbors.

Shannon's formula assumes AWGN, but the Theoretical section illegally considered interference to be the same as background noise. So the following simple experimental illustration investigates whether interference appear as white noise to the receivers.

If competing 802.11 signals looked like white noise, predicting packet delivery would be a relatively straightforward function of signal difference and it could simplify the design of MAC protocols. To understand how competing signals affect packet delivery, an experiment was performed where every pair of nodes in the network send broadcast packets for 10 seconds. All CCA mechanisms are turned off, so the nodes sent regardless of whether they heard other traffic on the channel or not.

Figure 16 shows the result of what happens when two nodes send at the same time. This figure shows the delivery probability of the more powerful signal versus the signal difference of the two signals. For each point in the graph, each of the two senders sent for 10 seconds one at a time to measure each sender's signal at all the receivers. After this was completed, the two senders transmitted concurrently for 10 seconds and the receivers recorded which packets they received from which sender.

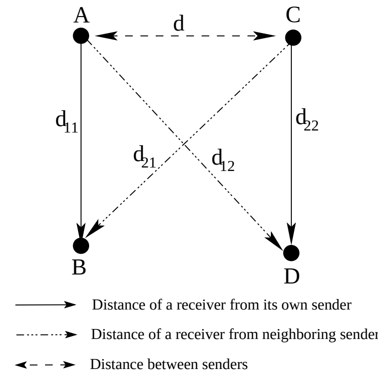

Figure 3: Example network where nodes A and C wish to send to nodes B and D.

For 6 and 12 mb/s, these graphs look similar to Figure 15: when one sender has a over 10 dB more signal, the higher signal will be received. For smaller differences (less than 5dB) it is difficult to decide which sender the receiver will hear. Figure 16 supports the claim that other 802.11 signals look like AWGN to the receiver; if it did not, the points in Figure 16 would not look like those in Figure 15 and would be shifted much further to the right since receivers would need a larger signal gain to overcome destructive interference from the other 802.11 signal.

3.2 Ideal radio model

Section 4 assumes radios to operate near the maximum throughput as defined by Shannon's capacity theorem. According to the theorem, it is possible to achieve error-free transmission at a maximum rate of T bits per second, over a channel with bandwidth B, given a signal of power S and additive white Gaussian noise of power N at the receiver:

$$T = B \cdot \log_2(1 + SNR) \tag{1}$$

where, SNR is the signal to noise ratio. Band width of the channel, B is assumed to be 1Mbps, throughout Section 4. The signal attenuation from path loss between the sender and receiver is $\frac{1}{d^k}$, where d is the distance from the sender in meters. Earlier studies of indoor propagation have found path loss coefficient k between two and four. We use $k = 2$ throughout the discussions in this paper. The free-space model simplifies analysis, and underestimates attenuation, since obstacles frequently attenuate interference in the real world. More realistic propagation is considered by the experiments in Section 5.

The signal-to-noise ratio in Equation 1 is a power ratio, where the signal and noise terms are in milliwatts not decibels. The remaining radio parameters reflect those of a typical 802.11a hardware. The channel bandwidth is 16.6MHz and the background noise is -95 dBm, which is the typical thermal noise found in an office environment.

4. THEORY

This section derives the expression for the optimum transmit power level combination for the nodes of a two-sender network. To better understand the theory, an intuitive explanation of why the theory should hold in practice is provided using results from simulations. Figure 3 shows an example of a network with two sender-receiver pairs where nodes A and C transmit to nodes B and D, respectively. The following notations are used.

- P_1 - Transmit power of sender A (AB is the sender-receiver pair 1)
- P_2 - Transmit power of sender C (CD is the sender-receiver pair 2)
- P_{max}-Maximum transmit power level of the nodes.
- d_{11} - Distance between sender A and receiver B
- d_{22} - Distance between sender C and receiver D
- d_{12} - Distance between sender A and receiver D
- d_{21} - Distance between sender C and receiver B
- N - background noise
- T_1 - Total throughput of nodes A and B
- T_2 - Total throughput of nodes C and D
- T - Total throughput of the network

All power levels, including the background noise level, are expressed in milliwatts. The Signal to Interference and Noise Ratio (SINR) at the receivers B and D can be calculated as follows.

$$SINR\ at\ B = SINR_1 = \frac{\frac{P_1}{d_{11}^2}}{\frac{P_2}{N+d_{21}^2}} \tag{2}$$

$$SINR\ at\ D = SINR_2 = \frac{\frac{P_2}{d_{22}^2}}{\frac{P_1}{N+d_{12}^2}} \tag{3}$$

$$\tag{4}$$

Hence the total throughput of the network is given by
Throughput of sender 1=$B * \log_2(1 + SINR_1)$
Throughput of sender 2=$B * \log_2(1 + SINR_2)$
Total throughput of the network is

$$T = B \cdot \log_2(1 + SINR_1) + \log_2(1 + SINR_2) \tag{5}$$

The following discussion proves that adaptive power control does not improve the throughput of any two-sender wireless network.

4.1 Expression for optimum transmit power levels

Considering Equation 5 as a function of two variables P_1 and P_2, one of the optimum values for (P_1, P_2) combination that maximizes T would be $P_1 = \infty, P_2 = \infty$. The following analyzes the equation a bit differently to show that this point $(P_1 = P_2 = \infty)$ corresponds to $P_1 = P_2 = P_{max}$, since there is always a maximum bound on the transmit power levels of the nodes.

Let Shannon's equation be re-written as follows to reduce the number of variables

$$T = B \cdot \log_2(1 + \frac{P_1}{M_1 + P_2K_1}) + \log_2(1 + \frac{P_2}{M_2 + P_1K_2}) \tag{6}$$

where, $M_1 = Nd_{11}^2$, $M_2 = Nd_{22}^2$, $K_1 = \frac{d_{11}^2}{d_{21}^2}$, and $K_2 = \frac{d_{22}^2}{d_{12}^2}$. Let power level P_2 of sender 2 be kept a constant. The

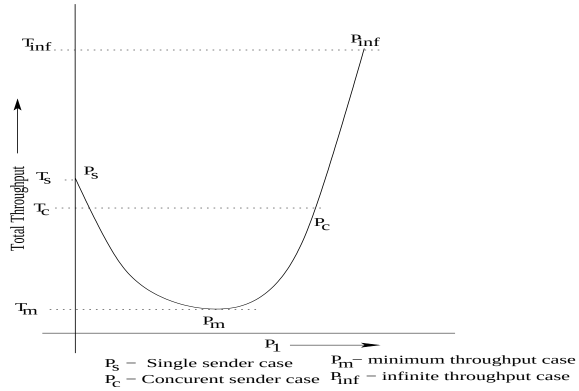

Figure 4: Graph representing P_1 versus T for a given P_2, N, and topology

throughput T is thus a function of P_1, given the topology, background noise, and power level of sender 2. The expression in equation 6 is thus a quadratic function in P_1 resulting in an 'U' shaped curve as shown in Figure 4, with one or two maximum values and one minimum value. Solving equation 6 for P_1, the maximum value can be found to occur at $P_1 = \infty$ or $P_1 = 0$ and the minimum is found to occur at

$$P_m = P_1 = \frac{-M_2 \pm \sqrt{(P_2^2K_2K_1 - M_2P_2 + P_2K_2M_1)}}{K_2} \tag{7}$$

Figure 4 shows how these points might be located in the graph for Equation 6. Here, P_s, P_{inf} correspond to the power levels of sender 1 (P_1), which are two end-points of the curve represented by the quadratic equation 6. P_s correspond to $P_1 = 0$ that is the single sender configuration where only sender 2 transmits with power level P_2 and sender 1 is switched off. P_{inf} denotes the point $P_1 = \infty$ and P_2 is a fixed value. The behavior of this graph can be explained as follows.

Initially, when $P_1 = 0$, only sender 2 transmits at a time with power P_2 and $T = T_s$. When sender 1 is switched on and as P_1 is increased from 0, throughput of the sender-receiver pair 1 increases from 0 and throughput of the old sender-receiver pair 2 decreases slowly due to increase in interference from sender 1. Since, $P_1 < P_2$, the net throughput decreases. However, after P_1 reaches a certain value, since P_2 is kept constant, the increase in throughput observed by receiver 1 becomes much higher than the decrease in throughput experienced by receiver 2. Thus, after the point P_m, the total throughput of the network starts increasing with P_1. Sometimes, even a small value of P_1 may increase the throughput of the network, in which case a '/'-shaped curve might be observed with $T_s = T_m$.

Let P_c denote the point $P_1 = P_2$. The following 2 claims prove that adaptive power control is not useful for two-sender networks.

- Claim 1: Since there is always a maximum bound on the power, P_{inf} is not possible and thus one of the end points of the 'U' or '/' curve becomes $P_1 = P_{max}$. The claim here is that, this $P_1 = P_{max}$ should also be equal to P_2 and represents the point P_c. Thus the two end points of the 'U' or '/'-shaped curve are P_s and P_c (single or common)

- Claim 2: The theory also claims that depending on whether T_s is greater or lesser than T_c, single or common transmission with maximum power is always the best case, which means that minimum point in the quadratic curve never occurs at P_c ($T_m < T_c$), in other words since P_2 is fixed $P_m < P_c$.

  However, minimum may occur at P_s ($P_s = P_m$), in which case, the maximum throughput will be observed at P_c, which results in '/' curve. It is always a choice between P_c and P_s which is the best strategy.

A detailed explanation and proof for the above claims and a simple example is provided in the appendix.

Since these claims are theoretically shown to be true (see appendix), the above discussion has proved that it is always one node (single transmission) or both the nodes (c common transmission) transmitting with maximum power level is the best strategy for any two-sender wireless network.

4.2 Classification of two-sender topologies

Two sender networks basically have two kinds of behavior and it is easier to understand these behavior by classifying the network appropriately.

1. *Theoretical definition:* Any two-sender topology belongs to either *sparse* or *dense* topology based on the characteristics defining the topology (d_{11},d_{22}, d_{12}, d_{21}), power levels of the two nodes (P_1, P_2), and the background noise, N. Let "P_{cs}" be defined as follows.

$$P_{cs} = \frac{-b \overset{+}{-} \sqrt{(b^2 + 4K_1K_2M_1)}}{2K_1K_2} \tag{8}$$

   where $b = (K_1M_2 - K_1M_1 - M_1)$.

   - Case 1: If $P_{max} < P_{cs}$ and $P_1, P_2 \leq P_{max}$, the topology is sparse. For a sparse topology, both the nodes transmitting concurrently with maximum power is the best strategy. That is, choosing $P_1 = P_2 = P_{max}$ is the optimum strategy.
   - Case 2: If $P_{max} > P_{cs}$ and $P_{cs} < P_1, P_2 \leq P_{max}$, the topology is a dense topology. In this case, one node sending at a time with maximum power level provides the maximum throughput for the network. That is, $P_1 = P_{max}$ and $P_2 = 0$ or $P_2 = P_{max}$ and $P_1 = 0$ depending on whether T_1 is greater or lesser than T_2, respectively.
   - Case 3: If $P_{max} = P_{cs}$, both single and concurrent transmissions with maximum power provides equal and maximizing throughput and thus either choice is best.

2. *Intuitive definition:* In other words, a sparse topology is one in which increasing the transmit power level of both the nodes P_1 and P_2, such that they are less than P_{cs} and P_{max}, increases the total throughput more than just increasing the throughput of one sender. This is because in these networks, increase in throughput due to increase in signal is higher than decrease in throughout due to increase in interference. Thus the net throughput always increases. Such networks are characterized either by node pairs separated well-enough from each other or the presence of very high background noise, or transmission power being less or a combination of all three such that $P_{max} < P_{cs}$. Note that increasing noise, decreasing the distance between node pairs, and decreasing the transmission power level of the nodes, all have the same effect on throughput of a network.

   Similarly, a dense network is one in which increasing power level of both the nodes simultaneously does not increase the throughput of the network as much as increasing the transmit power level of one sender. In such cases, single transmission with maximum power is always the best choice.

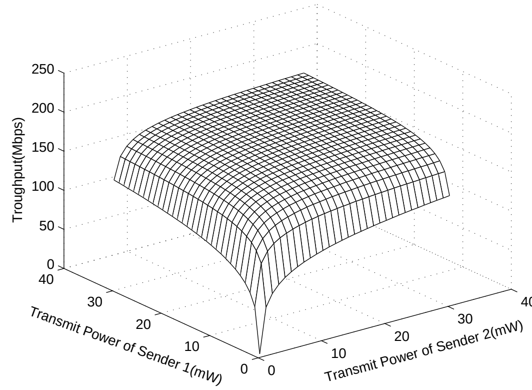

Figure 5: Surface plot representing total throughput as a function of P_1 and P_2 for a sparse square topology

These cases will become clear in the following discussion, where each topology is discussed both theoretically and intuitively.

4.3 A Sparse Topology

The sparse topology, examined in this section, is a setup with senders and receivers positioned to the corners of a 1x10 rectangle as shown in Figure 3, such that each sender is one unit of distance from its receiver ($d_{11} = d_{22} = 1$), and ten units from the other transmitter($d = 10$). Figure 5 plots the total throughput of this topology as a function of P_1 and P_2 which are each varied from 0mW to 30mW. A background noise level of 0.1 mW is chosen.

For this topology, if P_1 is maintained at P_{max}, increasing P_2 0 to P_{max} gives a '/'-shaped curve, which can be observed from Figure 5 and thus in this case, $P_c = P_{max}$ gives the maximum throughput.

Figure 5 shows the throughput surface to be concave and symmetric around the line where $P_1 = P_2$ due to the symmetric nature of the topology. Symmetric topology means that the distance from sender to receiver is same for both the pairs ($d_{11} = d22$) and the distance from the neighboring sender to the unintended receiver is the same for both the pairs ($d_{12} = d_{21}$). Assuming that transmitters are bound to transmit at a power level of 30 mW or less, we observe that the choice of power level for nodes A and C that maximizes the throughput of the network is to transmit at 30mW concurrently. The fact that they transmit concurrently is essential for maximizing throughput; notice that as we move

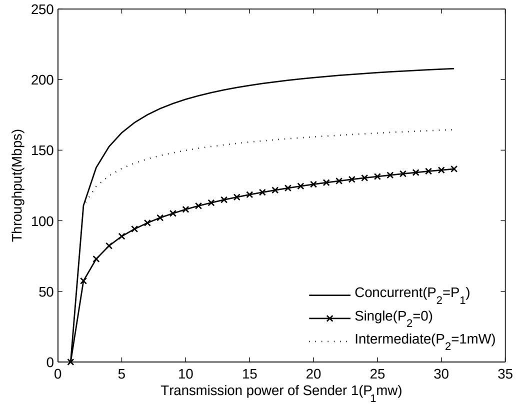

Figure 6: Comparison of total throughput when $P_2 = P_1$ and $P_1 = 0$ for single, concurrent and power controlled transmissions for a dense square topology.

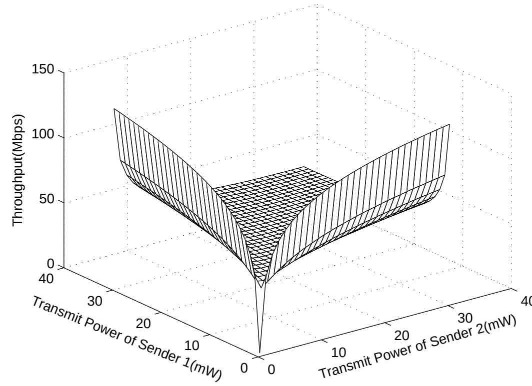

Figure 7: Surface plot representing total throughput as a function of P_1 and P_2 for a dense square topology

along either of the $P_1 = 0$ or $P_2 = 0$ axes, we do observe an improvement in throughput, but it is not as significant an improvement as moving along the $P_1 = P_2$ line. This relationship between single and concurrent can be observed from the two dimensional reduction of of Figure 5 shown in Figure 6. The intuitive reason this occurs is that for this topology and given transmit power, the senders can transmit concurrently since the interference experienced by receivers is less significant compared to the background noise of the network. Having a higher background noise has similar effects as moving the sender-receiver node pairs farther apart or operating in a relatively low power compared to the background noise and distance of the node pairs.

Another important observation is that the $P_2 = P_1$ and $P_2 = 0$ lines actually form the the maximal and minimal achievable throughput curves for different values of P_1 for the maximum power level of 30mW. Figure 6 illustrates this for the sparse topology at different P_2 values such that $0 < P_2 < P_1$, here we assume the intermediate power level of $P_2 = 1mw$. For the sparse topology, none of these curve outperforms $P_2 = P_1$ and no curve under performs $P_2 = 0$. For later examples, we will see that $P_2 = 0$ can outperform $P_2 = P_1$, but for all values of P_1 one of the lines $P_2 = 0 = P_{max}$ (single transmission) or $P_2 = P_1 = P_{max}$ (concurrent transmission) will always form the maximum throughput curve.

4.4 A Dense Topology

This section examines a dense topology. In this topology, nodes are arranged in a square with sides that are 1 unit of distance in length such that each sender is both 1 unit of distance from its respective receiver and the other sender. That is in Figure 3, $d_{11} = d_{22} = d = 1$.

Theoretically, we observe a 'U' shaped curve in Figure 7 with T_s always greater than T_c. Thus, $P_1 = P_{max}, P_2 = 0$ or $P_1 = 0, P_2 = P_{max}$ is the best strategy.

We see a markedly different situation in Figure 7 when compared to Figure 5. Again, due to the symmetric nature of our topology, the plot is symmetric around $P_1 = P_2$ line, but the dense topology results in a convex throughput surface instead of the concave throughput surface resulting

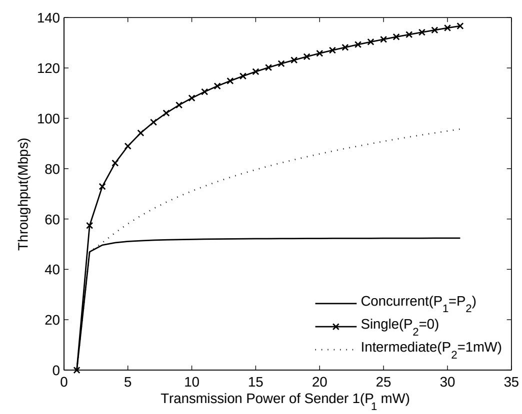

Figure 8: Comparison of total throughput when $P_2 = P_1$ and $P_1 = 0$ for a dense square topology.

from the sparse topology. Here we see that moving along the $P_1 = 0$ and $P_2 = 0$ axes produces a much higher throughput than moving along the $P_1 = P_2$. Figure 8 better illustrates the difference between choosing a common power and allowing one sender to transmit at a time. Having senders transmit one at a time using some kind of media contention or TDMA scheme is better than allowing nodes to transmit concurrently. This makes intuitive sense for a dense topology in which interfering signals have a significant greater impact on one another than the background noise level.

4.5 A Topology in Between

So far, we discussed the two extreme cases of sparse and dense networks. The throughput-maximizing power level is always the highest available but we make very different choices in sparse and dense networks. In sparse networks, nodes can transmit at the same time in order to achieve optimal throughput for the given topology mainly because increasing the power of the other neighbor node . In dense networks, its better for nodes to transmit one at a time.

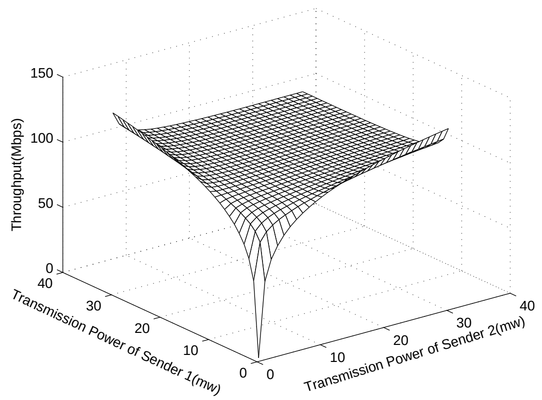

Figure 9: Surface plot illustrating total throughput as a function of P_1 and P_2 for a topology which changes from sparse to dense, as P_{max} crosses 10mW.

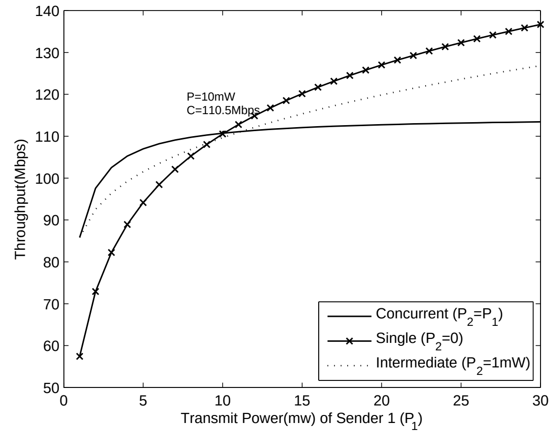

Figure 10: Comparison of total throughput when $P_2 = P_1$ and when $P_1 = 0$ for a topology with intermediate density for a given power range (0 to 30mW) and background noise (0.1mW).

Now let us examine a topology in which as sP_{max} increases, the topology moves from sparse to dense. We again examine a rectangular topology except that we now shorten the distance between A and C of Figure 3 to 3 ($d_{11} = d_{22} = 1$ and $d = 3$). For the same power levels as that of the sparse and the dense, this topology will look sparse till $P_{max} = P_{cs}$ and then switches to dense topology. The background noise is 0.1 mW.

Figure 10, shows two lines corresponding to single and concurrent transmissions where both the nodes operate at same power level P. Interesting point in this figure is that all other power combinations are well below concurrent line on the left of the point of intersection of the concurrent and single ($P_{cs} = P_{max}$) and below single on the right of P_{cs}, proving that power control has no effect. Initially concurrent transmission with common power performs better than single transmission until the power levels of the nodes reach a certain value. Here this break-even point is $10mW$. Thereafter, throughput due to single transmission is greater than that of concurrent transmission.

Theoretically speaking, in figure 9, when $P_{max} < 10mW$, we can observe a '/'-shaped curve similar to the sparse topologies, for $P_1 = P_{max}$ and varying P_2 from 0 to P_{max}. Thus, common is better. When $P_{max} > 10mW$, we can observe a 'U'-shaped curve and here since $T_s > T_c$, we have single transmission with maximum transmit power to be the best strategy.

The reason behind this is as follows. Initially, when only one of the sender nodes transmit, the throughput is given by Equation 1. When adding a second sender, one of two things happen: Either the throughput of the second sender increases the total throughput of the system or the interference caused by the senders at the other receivers is so high that it reduces the overall throughput.

Similarly, increasing the power level P of both senders not only increases the signal at the receivers but it also increases the interference contributed from the other sender.

Initially when the transmit power of the nodes in Figure 3 are low, adding a second sender increases the signal more than the interference it causes because the background noise is more significant than the interference from the other node. However, when power levels of the nodes are higher, interference becomes significant; single transmission gets more throughput than concurrent transmission.

4.6 Asymmetric Topologies

The topologies examined thus far are arranged symmetrically (i.e., $d_{11} = d_{22}$ and $d_{12} = d_{21}$ in all the topologies). This has resulted in the total throughput surface plots symmetric around the $P_1 = P_2$ of Figures 5, 7, and 9. Certainly, we would be hard pressed to find a real wireless network in which this strict regimen of symmetric spacing is realistic. Therefore, understanding the theoretical conditions of asymmetric topologies becomes important.

In asymmetric topologies constrained such that the intended signal strength is always much higher than sum of interference and noise, the behavior is largely like that of sparse and dense symmetric topologies. Interfering signals never arrive at a receiver with a greater power than the intended signal. The Shannon model correctly predicts behavior for such cases, and the results are very similar to those for the topologies examined thus far.

If an interfering sender is closer to the receiver than the intended sender, its signal will always have greater power than that of the intended sender. In the face of simultaneous transmissions at a common power, the interfering signal's power will exceed the intended signal's power and lead to an extremely diminished throughput. According to the Shannon capacity theorem, it should still possible to communicate some information despite the $SINR < 1$; yet the throughput will be close to zero.

However, in real 802.11 wireless networks, as explained in Section 3, as long as the intended signal strength is atleast 6dB higher than the sum of interfering signal and background noise, the receiver will be able to decode the signal correctly, nevertheless the throughput achieved being proportional to this difference. Fortunately, while it does not capture the exact behavior of 802.11 devices, the Shannon capacity theorem is nevertheless close enough and leads to choice between single and simultaneous transmissions as il-

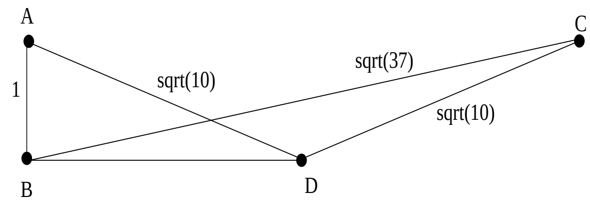

Figure 11: An asymmetric topology

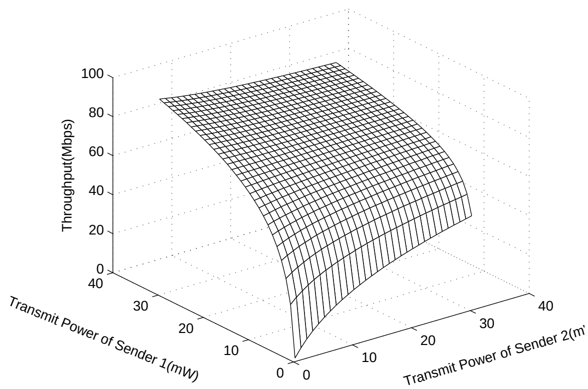

Figure 12: Surface plot illustrating total throughput as a function of P_1 and P_2 for a T-shaped topology.

lustrated by the following example. Figures 12 and 13 show the throughput achievable by nodes in an asymmetric topology as shown in figure 11 with $N = 0.1mW$. These figures look very similar to the corresponding Figures 5 and 6 detailing a sparse topology. A noteworthy characteristic of this topology is that A is as close to D as C is. In such a topology of 802.11 standard nodes, simultaneous transmissions would result in the A-to-B transmission being received at D with same strength as C. According to the Shannon capacity theorem, the C-to-D channel's throughput would be very small and close to zero under the same conditions. In real network, this will be exactly zero.

Theoretically, when P_{max} is lesser, we can observe a sparse topology. That is, fixing P_1 and changing P_2 and fixing P_2 and changing P_1 gives a '/' shaped curve and common transmission is best in this case. However, as P_{max} increases, fixing P_1 and varying P_2 provides a 'U'-shaped curve with single transmission ($P_2 = 0$) being the best choice. For a given optimum P_2 (which is zero in this case, as opposed to P_{max} in the case symmetric sparse topology), we can observe a '/'-shaped curve as P_1 is varied, suggesting that $P_1 = P_{max}$ is the best choice. Combining these two options, we have $P_1 = P_{max}$ and $P_2 = 0$ to be the best strategy.

This is clear from Figure 13. Intuitively, this makes sense as the very small throughput predicted by the Shannon theorem closely approximates the zero throughput that is actually experienced.

5. EXPERIMENTS

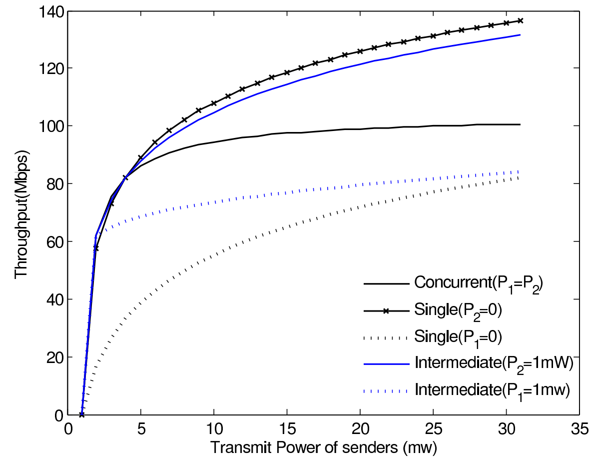

Figure 13: total throughput as a function of transmission power P_1 where $P_2 = P_1$ for a T-shaped topology.

The rest of the paper will verify the conclusions from the theoretical results presented earlier in this paper. This section provides the results to support the theoretical conclusions for the two-sender case. Before understanding these results, it is worth studying the limitations of the present wireless radios through a simple single sender-receiver pair network.

5.1 Experimental Setup

This subsection describes the network used to conduct the experiments. A simple two-sender network consists of two sender-receiver pairs placed in sparse and dense topologies similar to Figure 3. As defined previously in Section 4, dense and sparse are defined relative to each other in the sense that the separation between the two sender-receiver pairs is higher for a sparse topology than the dense topology. *Click Modular Router* is used to control the transmission of the wireless node. All nodes use identical 802.11 a/b/g cards based on the Atheros 5213 chip-set. Except as noted, for the measurements described in this paper the cards transmitted in 802.11a mode, in the 5 GHz band. The devices use standard 5 dBi omni-directional antennas.

In each measurement experiment, both the sender nodes send 2000-byte broadcast packets as fast as possible, while both the receivers passively listen and filters out only the packets destined to them. The senders log each packet sent, and all the other nodes log which they receive. Because the packets were sent in broadcast mode, there were no acknowledgments or retransmissions. All mechanisms provided by the hardware for clear channel assessment were disabled, so the senders transmitted packets at full rate, regardless of the channel conditions they observed.

This means the data reflect true concurrent transmissions, and are not skewed by deferrals. During the experiments, there was no other observed 802.11a traffic. For a given combination of senders, all senders transmit 2000-byte broadcast packets for 10 seconds. This was repeated for each bit-rate at different power levels. For each experiment, the receiver logs the total number of packets received from its own sender (that is after filtering out the packets received

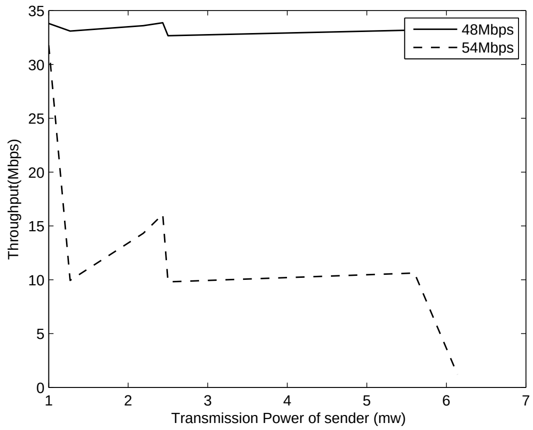

Figure 14: Single-sender throughput ilustrating receiver overloading at higher bit-rates and power levels.

from the neighboring sender). The throughput for an experiment is the ratio of number of packets received to the duration of the experiment (10 seconds). The total throughput of the network for an experiment is the sum of the individual throughputs of both the receivers.

5.2 Limitations of wireless receivers: study of single-sender network throughput

One would expect the throughput of a single sender network to increase as the power level of the sender increases for a fixed bit-rate. It would be surprising to know that, in reality, the behavior is completely different because of the limitations of the hardware.

Figure 14 shows the single-sender throughput for the bit-rates 48 and 54Mbps. As seen from the figure, for 54Mbps, the throughput actually decreases with increase in power. Infact, theoretically, this should have been the reverse. Even 48Mbps, exhibits similar behavior but seems to be less affected. These are because of the effects of *receiver overloading*, which occurs when a closely placed sender transmits at a higher bit-rate and higher power level. A good hardware designed reduces this effect.

Another limitation of the 802.11 radios for higher bit rates of around 54Mbps, the maximum power levels at which the 802.11 cards can operate reduces slightly and varies for radios design by different manufacturers.

All the experimental results provided in the subsequent discussions avoid transmission of packets at very higher power levels to reduce these effects. Yet, the receiver overloading cannot be eliminated at all and this effect increases in the presence of interference in the two-sender case.

5.3 Interference from other senders

Shannon's formula assumes AWGN, but the Theoretical section illegally considered interference to be the same as background noise. So this section investigates experimentally whether interference appear as white noise to the receivers.

If competing 802.11 signals looked like white noise, predicting packet delivery would be a relatively straightforward

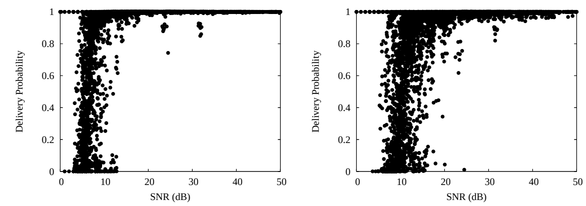

Figure 15: Delivery probability for single senders versus SNR for 6 and 12 Mbps.

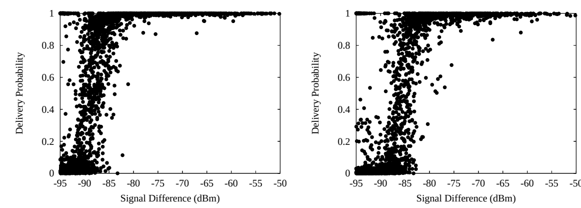

Figure 16: Delivery probability versus signal difference when two nodes are sending. The signal difference required for delivery at 6 and 12 Mbps are similar to those in Figure 15, suggesting that other 802.11 signals do not destructively interfere.

function of signal difference and it could simplify the design of MAC protocols.

To understand how competing signals affect packet delivery, an experiment was performed where every pair of nodes in the network send broadcast packets for 10 seconds. All CCA mechanisms are turned off, so the nodes sent regardless of whether they heard other traffic on the channel or not.

Figure 16 shows the result of what happens when two nodes send at the same time. This figure shows the delivery probability of the more powerful signal versus the signal difference of the two signals. For each point in the graph, each of the two senders sent for 10 seconds one at a time to measure each sender's signal at all the receivers. After this was completed, the two senders transmitted concurrently for 10 seconds and the receivers recorded which packets they received from which sender. For 6 and 12 mb/s, these graphs look similar to Figure 15: when one sender has a over 10 dB more signal, the higher signal will be received. For smaller differences (less than 5dB) it is difficult to decide which sender the receiver will hear.

Figure 16 supports the claim that other 802.11 signals look like AWGN to the receiver; if it did not, the points in Figure 16 would not look like those in Figure 15 and would be shifted much further to the right since receivers would need a larger signal gain to overcome destructive interference from the other 802.11 signal.

5.4 Two-sender Experimental Results

The two-sender experiments were conducted for two different topologies: dense and sparse networks. Figure 17 shows the two-network dense topology used in the study. The results are aimed at analyzing the throughput for various power levels of the the two senders. At any given power level of the two senders, experiments are conducted

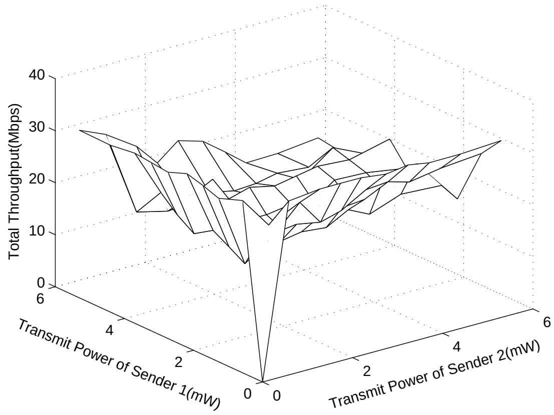

Figure 17: Total throughput for dense network with optimum power level ($p_1 = 4mW, p_2 = 0$

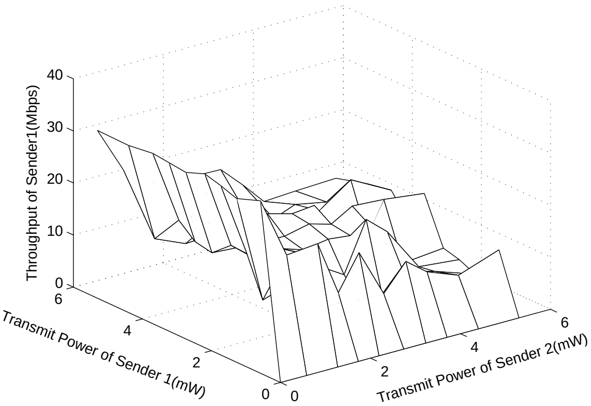

Figure 18: Throughput of sender 1 in a dense network

for all bit-rate combinations of the two senders and maximum throughput achievable for the best bit rate combination is the throughput for that given power level combination of the two senders. Figures 17, 18, and 19 shows the total throughput and throughput of senders 1 and 2, respectively for various power levels. Here, the power levels used are low in order to avoid operating in the receiver overloading power range. Since this is dense network, as we increase the maximum available power level, the maximum throughput is achieved when the nodes transmit individually. This is an expected behavior that matches with the theoretical claims.

However, an interesting point to be noted here is that for any given maximum power level of the nodes, though the maximum throughput is achieved for a single sender case, an intermediate power level rather than maximum power level proves to be optimum. For example, for the dense topology under study, the maximum throughput is achieved when node 2 sends at around 5mW, but the best for this network is around 4mW. This is due to receiver overloading.

Similarly, figures 20, 21, and 22 shows the total throughput and throughput of senders 1 and 2, respectively for a much sparser topology than the denser topology defined earlier. The maximum throughput is achieved for a common power level, which is theoretically expected from a sparser network. However, due to the receiver overloading, the common power at which this happens is an intermediate power of 2mW.

Though the experiments match with theory, the experiments conducted are not exhaustive of all two-sender topologies. Receiver overloading makes it even more complicated to prove. Till now, it was assumed that power control was important for a two-sender network because increasing the power increases interference and will not allow concurrent transmission. However, it is clear that both from the perspective of theory and experiments, the need for power control is to be reexamined and in a newer perspective than based on interference and transmission range.

6. FUTURE WORK

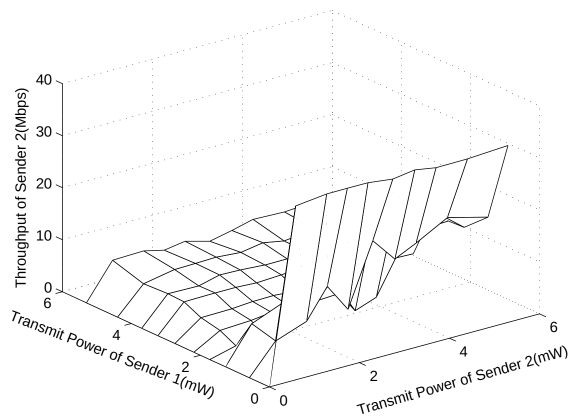

Figure 19: Throughput of sender 2 in a dense network

This section discusses some of the future possibilities for the work presented in this paper.

6.1 Benefits of Power Control for n-sender networks

One of the easiest ways to increase throughput for a network with single sender is to increase the signal to noise ratio by having the sender transmit at a higher power level. A natural extension of this paper would be analyze throughput benefits for n sender case. However, when there are multiple senders transmitting concurrently, increasing the transmit power of a single node will increase the noise level from the perspective of all the other senders. In some cases when all the nodes increase their transmission power, increase in a node's signal may be very less compared to the increase in the total interference contributed by increase in the transmission power of all the neighboring senders. It is not clear whether increasing the transmission power of the nodes increases the overall throughput of the network. The following discussion has some analysis for n-sender case to

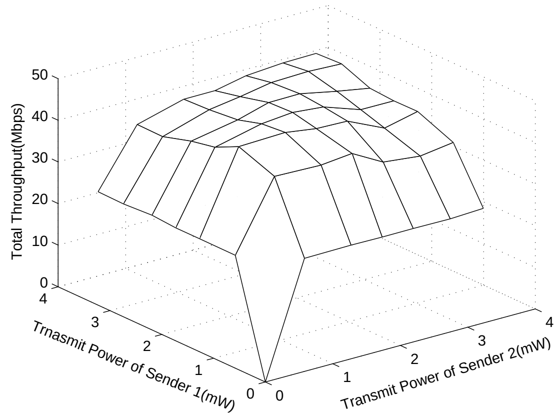

Figure 20: Total throughput for sparse network with optimum power level ($p_1 = 2mW, p_2 = 2mW$)

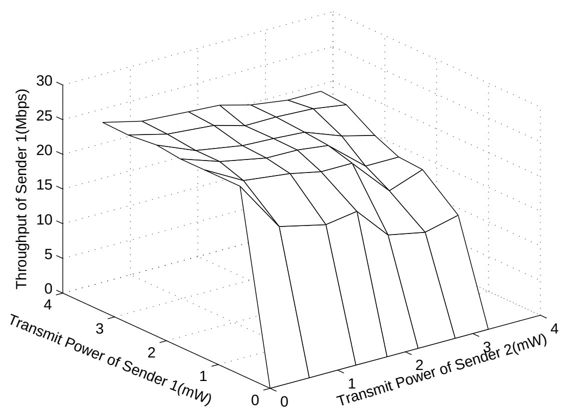

Figure 21: Throughput of sender 1 in a sparse network

motivate re-examination of benefits of power control for any wireless networks.

The basic step to understanding n-sender case would be to see if there are benefits of increasing the power of all the nodes by a same amount. Figure 23 shows the throughput versus the transmit power levels at N=-95dBm. The figure shows that the throughput increases as P and levels of for higher P. This is because for a dense network with negligible N, Shannon's equation for the n-sender case is as follows.

$$C_n = B \cdot \sum_{i=1}^{n} \log_2(1 + \frac{(\frac{1}{d_{ii}^k})}{\sum_{j=1,j\neq i}^{n} \frac{1}{d_{ji}^k}}) \tag{9}$$

Since P cancels out in the above equation. That is, increasing the power increases both the signal and aggregate interference by a same amount and thus the SINR remains the same, no matter whichever power level is used.

However assuming $N = 0$ is practically incorrect. In the presence of background noise, the increase in power level of

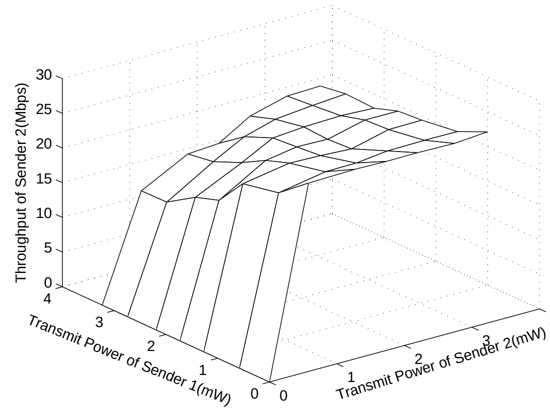

Figure 22: Throughput of sender 2 in a sparse network

both the nodes, increases the throughput till some power level is reached, after which, since the transmit power level is very high compared to the background noise, the network behaves similar to the case where background noise is zero and thus throughput remains constant. This conclusion has a serious effect on the well-known assumption used by FTR model. Thus the above discussion makes clear that, the FTR model's implication that decreasing power will increase total throughput is not correct for these radios.

Since the two sender results convey that single transmission is better for dense networks and concurrent is better sparse networks, it is intuitive to say that for an n sender network with varying densities (which may arise due to maximum possible power level of he nodes, varying background noise, distance between the nodes), grouping the nodes based on density and deciding between simple single and concurrent for each group should give a maximum throughput than using complex power control protocols with intermediate power levels.

6.2 Analyzing wireless networks

Many assumptions have been made to model the wireless networks, such as the concept of transmission range and the protocols designed based on some of these assumptions without understanding how accurate or approximate they are, may fail in practice.

Thus a new perspective is required on not only understanding the benefit of power control but also how factors such as power, noise, and topology affect the wireless network capacity both individually and jointly.

While the analysis for a simple two-sender power control is so complex, it is not surprising to realize the complexities that would spring from analyzing the effect of all the factors that affect the performance of wireless network.

7. CONCLUSIONS

The ability of different senders to re-use the same spectrum in different areas is perhaps the most important ingredient in the total throughput of large access-point and mesh wireless networks. It is well known that total throughput is proportional to the total area of the network. This pa-

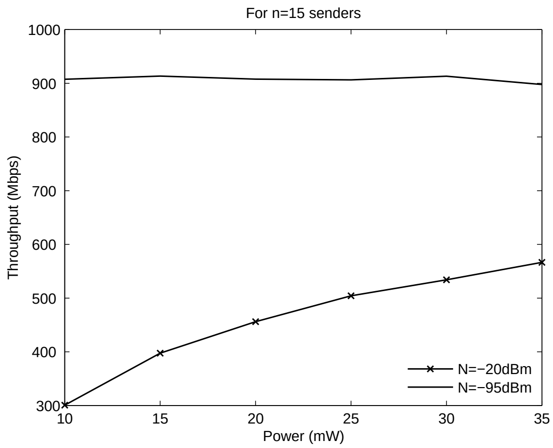

Figure 23: Throughput versus the number of nodes at two different power levels. Increasing the power level has no affect on capacity because the noise levels at the receiver increased in proportion to the signal.

per analyzes the other factors that determine the amount of throughput potentially available from re-use, with particular attention to how 802.11-like radios would have to be managed in order to achieve that potential throughput.

The main findings, derived from application of Shannon's capacity theorem, are as follows. There is no potential in network throughput on applying power control to a two sender network. It is typically best for many nodes to send concurrently, since the gain in throughput from more senders offsets the loss in throughput from more interference. Adaptive transmit bit-rate is critical to maximize throughput, since interference levels from multiple concurrent senders may be high and different signal-to-noise ratios require different bit-rates to maximize per-sender throughput. Transmit power control is important in that the power level must be high enough that received signal strength is above background noise, but further increases in power level (by all nodes) do not help or harm total throughput. One implication of these results is that surprisingly high levels of re-use should be possible, in the sense that senders that can hear each other can nevertheless profitably send at the same time. The paper verifies that result on 802.11 test-beds. Some interesting facts about limitations and complexities of the hardware were also analyzed.

8. REFERENCES

[1] Aditya Akella, Glenn Judd, Srinivasan Seshan, and Peter Steenkiste. Self-management in chaotic wireless deployments. In *MobiCom '05: Proceedings of the 11th annual international conference on Mobile computing and networking*, pages 185–199, New York, NY, USA, 2005. ACM Press.

[2] A. Behzad and I. Rubin. Multiple access protocol for power-controlled wireless access nets. *IEEE Transactions on Mobile Computing*, 3(4):307–316, 2004.

[3] P. Gupta and P. R. Kumar. The capacity of wireless networks. *IEEE Transactions on Information Theory*, 46(2):388–404, March 2000.

[4] W. H. Ho and S. C. Liew. Scalability of wireless network capacity with power control. In *Proceedings of IEEE Vehicular Technology Conference*, pages 710–714, 2004.

[5] Jinyang Li, Charles Blake, Douglas S. J. De Couto, Hu Imm Lee, and Robert Morris. Capacity of ad hoc wireless networks. In *Proceedings of the 7th ACM International Conference on Mobile Computing and Networking*, pages 61–69, Rome, Italy, July 2001.

[6] M. Frank M. Gerharz, C. D. Waal and P. Martini. Influence of transmission power control on the transport capacity of wireless multihop networks. In *Proceedings of IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'04)*, pages 1016–1021, 2004.

[7] Timothy J. Shepard. A channel access scheme for large dense packet radio networks. In *ACM SIGCOMM*, pages 219–230, 1996.

[8] R. Rozovsky R. S. Sreenivas V. Kawadia, S. Narayanaswamy and P. R. Kumar. Protocols for media access control and power control in wireless networks. In *Proceedings of IEEE Conference on Decision and Control 2001*, pages 1935–1940, 2001.

APPENDIX

- Proof for claim 1: Let P_2 be fixed to an intermediate value less than P_{max}. Solving equation 6 for P_1 provides a 'U'-shaped curve. Let the total throughput of the network corresponding to $P_1 = P_{max}$ be greater than the single-sender case $P_1 = 0$. Thus $P_1 = P_{max}$ is the maximum for this curve, for a given value of $P_2 < P_{max}$.
  Now, if the value of P_1 is fixed to P_{max} and same procedure is repeated to find optimum P_2, this should occur at $P_2 = P_{max}$, since the equation 6 is again quadratic in P_2 if P_1 is held as a constant. Thus, from these two observation, it is clear that for a fixed P_2, it is P_s and P_c, which are two end points of the U-shaped curve.
  Also, Equation 5 has the optimum value at $P_1 = \infty, P_2 = \infty$.
- Proof for Claim 2: $P_m < P_c$.

  Proof by contradiction: Let $P_m > P_c$. This implies from 6,

$$\frac{-M_2 \overset{+}{-} \sqrt{(P_2^2 K_2 K_1 - M_2 P_2 + P_2 K_2 M_1)}}{K_2} > P_2$$

$$-M_2 \overset{+}{-} \sqrt{(P_2^2 K_2 K_1 - M_2 P_2 + P_2 K_2 M_1)} > P_2 K_2$$

  Thus,

$$P_2 < \frac{-b \overset{+}{-} \sqrt{(b^2 - 4(K_2^2 - K_2 K_1) M_2^2)}}{2(K_2^2 - K_2 K_1)}$$

  where, $b = 2M_2K_2 + M_2 - K_2M_1$. Since $P_2 > 0$, R.H.S of Equation 10 should also be greater than zero. Thus, solving R.H.S of Equation 10, we get two conditions, $-4(K_2^2 - K_2K_1)M_2^2 > 0$ and $2(K_2^2 - K_2K_1) > 0$ (or both < 0) which results in $P_2 < 0$ and $P_2 > 0$. Since the conditions are contradictory, the initial assumption $P_m > P_c$ is false.
  Thus, $P_m < P_c$ is proved.

.1 Illustration using Simple Example

Consider a simple topology as shown in Figure 3, with $d_{11} = 1, d_{22} = 1, d_{12} = \sqrt{2}, d_{21} = \sqrt{2}$. For this topology the optimum power value can be obtained by substituting the known values in Equation 5. Assuming a background noise of $N = 0.1$, the optimal values of transmission power of the nodes is given by

$$P_1 = \begin{bmatrix} 9/2 + 1/2\sqrt{85 + 4\,{P_2}^2 + 8\,P_2} \\ 9/2 - 1/2\sqrt{85 + 4\,{P_2}^2 + 8\,P_2} \end{bmatrix}$$

Table 1shows the throughput of the network for fixed power level P_1 for sender 1 and varying the power of sender 2 from 0 to 10mW, trying to figure out, if there is any other power less than 10 for which the throughput attains maximum.

$P_2(mw)$	$Throughput(Mbps)$
0	110.5263
1.0000	109.6482
2.0000	109.4986
3.0000	109.5361
4.0000	109.6482
5.0000	109.7968
6.0000	109.9655
7.0000	110.1461
8.0000	110.3339
9.0000	110.5263
10.0000	110.7215

Table 1: Network throughput by fixing $P_1 = 10mW$ and varying P_2 from 0 to 10mW

The throughput as explained in Section 4 is a 'U'-shaped curve. The Table 1 shows that the maximum throughput for maximum power of 10mW is 110.7Mbps attained when both the nodes transmit at common and maximum available power of the two senders.

This example provides a proof for why the intermediate power levels do not provide maximum throughput. Analysis of this equation purely theoretically may provide many more interesting results. But this paper is not restricted to just derivations. Rather, what is more interesting is the intuitive reasoning for why power control is not important for two sender case and the above discussion is just a mathematical proof with a complex formula.