

A Communication Technique for Multipath Channels*

R. PRICE†, ASSOCIATE MEMBER, IRE AND P. E. GREEN, JR.†, SENIOR MEMBER, IRE

Summary—Application of principles of statistical communication theory has led to a new communication system, called Rake, designed expressly to work against the combination of random multipath and additive noise disturbances. By coding the Mark-Space sequence of symbols to be transmitted into a wide-band signal, it becomes possible at the receiver to isolate those portions of the transmitted signal arriving with different delays, using correlation detection techniques. Before being recombined by addition, these separated signals are continuously and automatically processed so as to 1) apply to each an optimum weighting coefficient, derived from a measurement of the ionosphere response, and 2) introduce in each an appropriate delay such that they are all brought back into time coincidence.

After a brief introduction, a functional description of the system is presented. There follows a review of the communication theory studies, which indicate that such systems have certain optimal properties. Details of design of an experimental prototype Rake system are followed by the results of limited field tests of this prototype. Conclusions and recommendations for future work are given.

I. INTRODUCTION

MULTIPATH is a troublesome condition in many communication channels; the signal proceeds to the receiver along not one, but many paths, so that the receiver hears many echoes having, in general, different and randomly varying delays and amplitudes. The most familiar example occurs in high-frequency communication via the ionosphere [1, 2].

Multipath has been a problem from the early days of short-wave radio communication. Its influence on a communication system is usually described in terms of two effects—selective fading, and intersymbol interference—of which one or the other may be of predominant importance in the particular communication system being discussed [9].

Selective fading has to do with the relative *rf* phases of the signals delivered to the receiving antenna via the various paths. At any one frequency, the total received signal is a vector sum of individually delayed signals, their relative phase angles depending on the frequency and the echo amplitudes and delays. Therefore, since the echo amplitudes and delays are time varying, one observes large variations of the received signal strength at a single frequency as a function of time, or of the strength at a given time as a function of frequency; the latter is termed “selective fading.”

Intersymbol interference is associated simply with the *time delay* between first and last significantly large echoes. If the modulation is rapid enough, the echoes appearing in this modulation will result in a jumbling

or smearing of the intelligence, regardless of the form of signal used.

Previous system design for effectively combating multipath disturbances might be considered “passive”; that is, the effort has mainly been directed toward minimizing the undesired effects without having actual knowledge of the multipath characteristic. The use of single-sideband transmission (with or without exalted-carrier reception) [3], synchronous AM reception [4], and various forms of diversity reception [5] as anti-selective fading measures represent this type of approach. Fading has also been attacked by using coding [6].

The intersymbol interference aspect of multipath was long ago recognized to place a limit on the rate at which digital information could be communicated with time-division schemes. The use of multiple subcarriers (“frequency division”) each having long symbol-waveforms to carry a fraction of the total information rate over multipath has been standard for many years, and has recently received additional impetus from new techniques which yield considerably greater efficiency of frequency spectrum utilization [7–8]. A method of extending frequency-division approach to transmission of analog information has also been proposed [10].

Two other techniques, of quite a different nature from those mentioned previously, are directed toward the actual or effective suppression of all but one dominant path. The first employs a complex, steerable antenna array [50], while the second uses frequency modulation [12]. The latter method appears to work under only certain conditions which, unfortunately, are seldom met in practice [11].

In contrast to the philosophy of the systems just enumerated, the system which is the subject of this paper performs a continuous, detailed measurement of the multipath characteristic. This knowledge is then actively exploited to combat the multipath effectively. Simply stated, selective fading is opposed by detecting the echo signals individually, using a correlation method, and adding them algebraically (with the same sign) rather than vectorially, and intersymbol interference is dealt with by reinserting different delays into the various detected echoes so that they fall into step again. For reasons that will be seen shortly, this approach has been dubbed the “Rake” system.

This system evolved from and is largely justified on the basis of the application of methods of statistical communication theory to the problem of communication through multipath disturbances. Soon after the idea was thus established, as often happens, a heuristic

* Original manuscript received by the IRE, August 8, 1957; revised manuscript received, November 29, 1957. The research in this document was supported jointly by the Army, Navy, and Air Force under contract with Mass. Inst. Tech., Cambridge, Mass.

† Lincoln Lab., M. I. T., Lexington, Mass.

physical interpretation of the system became apparent. We shall, in the interest of clarity, present the latter straightforward, functional explanation of the Rake system first, in Section II. Without detracting from its significance, we defer until Section III the more rigorous derivation, which indicates that such a system, in addition to making intuitive sense, has certain optimal properties. In Section IV, the details of the construction of an experimental Rake system are given, and in Section V some results obtained in tests of the system over a transcontinental circuit are presented. In the concluding section the advantages and present drawbacks of the Rake system in relation to conventional systems are discussed, and suggestions are made for future improvement and extension.

II. FUNCTIONAL DESCRIPTION OF THE RAKE SYSTEM

A. General Principles of Design

Suppose we have decided to build a radioteletype system that will detect separately, then add up, each of the multiplicity of delayed signals arriving as a result of multipath. The transmission will then have to be wide-band,¹ for otherwise its time waveform cannot possess sufficient detail to permit the waveform at one instant of time to be distinguished unambiguously from that at another. Naturally, the wider the bandwidth, the finer will be the time resolution.

In addition to the requirement of providing multipath resolution, the transmission must of course be capable of carrying information. In a teletype system, this is accomplished by transmitting at will two distinguishable waveforms, one representing the Mark baud (or signalling element),² and the other Space. A commonly-used system is frequency-shift keying (fsk), where two sine waves of slightly different frequency are employed [13]. By analogy, we shall employ two different wide-band waveforms for the Rake transmission. Then by proper treatment of the received signal we can isolate a narrow region of delay and select for demodulation only the signal lying within it.

The principle of the Rake technique can be explained by starting with a simple fsk system shown in Fig. 1, which will shortly be reinterpreted to permit the use of a wide-band transmission and the attainment of the desired delay-isolation. The Mark and Space local oscillators shown in the receiver may be viewed as refer-

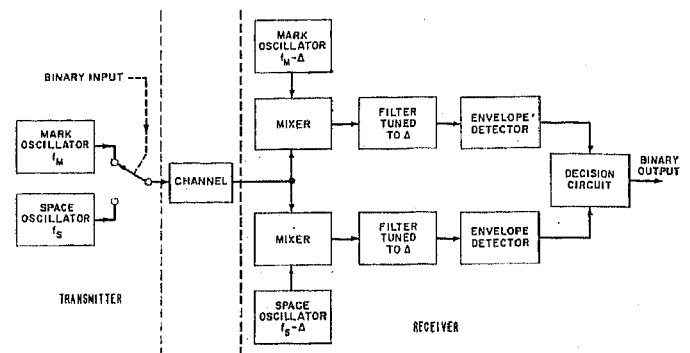


Fig. 1—Simple fsk system.

ence signals, with which the received signal is compared.

The comparison takes the form of determining (in the decision circuit) whether it is the envelope of the difference-frequency tone from the Mark filter or the Space filter that is the larger. The decision then represents one binary element of the output teletype sequence. Mixing (multiplying) the incoming and reference signals, and passing the difference-frequency result through a narrow-band filter is equivalent to the operation of *cross correlating* the two [16, 17] (that is, multiplying and then integrating).³ Since the decision is based on which symbol yields the larger correlation, it is apparent that for best performance the frequency-shift $|f_M - f_S|$ should be sufficient to yield uncorrelated Mark and Space waveforms [18].

A plot of the output of the filter as a function of relative delay between the reference signal and a copy of it arriving from the transmitter is the autocorrelation function of the reference (the Fourier transform of its power density spectrum) [17]. In the case of a reference having a continuous spectrum of nominal bandwidth W , the autocorrelation function has a single peak of width about $1/W$ seconds, centered at the origin, and disappearing toward zero elsewhere. Portions of the transmitted signal arriving with various delays relative to the appropriate reference appear in the output of the corresponding integrating filter as sine waves of amplitudes given by the values of the autocorrelation function of the reference at the respective delays. With fsk, W is small, the central correlation peak is very wide, and path contributions will appear in the filter outputs for a wide variety of delays. We have already discussed the destructive interference (selective-fading) that can result when contributions add with random phases.

When the receiver of Fig. 1 is modified by the substitution of wide-band transmitted and reference waveforms, the correlation function narrows very greatly, so that the correlator will make use of only those echoes which arrive within $1/W$ of synchronization with the references. Thus we need only make W sufficiently

¹ The inherent advantage of using a wide-band transmission in a multipath environment was first suggested by R. M. Fano in 1952 (private communication), who reasoned that it averaged out the selective fading. Subsequent channel-capacity analyses carried out on the fading justified this notion from the information-theory point of view [14]. An informal note by Fano [15] proposes another method of employing wide-band signals against multipath. Kharkevich [52] has independently suggested the use of wide-band signals as a measure against selective fading, but his receiver employs completely incoherent detection and this is comparatively inefficient.

² The word "baud," taken from teletype usage, will be employed frequently in this paper instead of "signalling element," or "bit."

³ A more detailed discussion of the relationship of this mix-and-filter scheme of correlation ("difference-frequency correlation" or "band-pass correlation") to the true mathematical operation of multiplying and integrating is given in [17].

wide to separate out the various echoes (this also corresponds to making W wide enough to "average out" the selective fading). Finally, we can be assured of making use of all the signals delivered to the receiver by the entire multipath structure, regardless of whether it consists of discrete paths or is a continuum, if we use a series of such correlators, each synchronized at successive delay increments of roughly $1/W$. Enough of them must be employed to span a region of delay sufficiently wide to encompass all echoes that are likely to appear. The name "Rake" seems an appropriate designation for such a scheme. Making each correlator synchronize at its assigned value of delay can be done by inserting the right amount of delay in either the reference or received signals. The latter has been chosen for several important reasons to be dealt with later.

With the above ideas in mind, it is now possible to set down almost completely an elementary block diagram of such a Rake receiver. This is done in Fig. 2. The reference sources, emitting the same Mark and Space waveforms that the transmitter uses (except for a frequency displacement of Δ_1), feed a series of correlators arranged along a delay line whose input is the received signal. Each correlator consists of a multiplier and an integrator, but since the correlator outputs should be added together, in order to make full use of each echo, we may either combine after separate integrations, or, what is equivalent, use a common integrator. The latter is obviously simpler, so, as the figure shows, the output of each Mark multiplier is added to that of the others on a "bus," and the sum is then passed into a common integrator. The same is done with the Space multipliers. A decision on whether Mark or Space was sent from the transmitter is made according to which of the two sums of correlations is the larger.

In order to be sure that the difference-frequency signals appearing in the buses all add constructively, their phases must be brought into common agreement. Furthermore, those multipliers responding to large paths should have their contributions to the buses accentuated, while those not synchronizing with any significant path, and thus being affected mainly by the channel noise, should be greatly suppressed, in order to increase the snr and consequently decrease the probability of making a decision error. More precisely, we may invoke the well-known result [19] that the maximum snr of a weighted sum, of which each term is the combination of a signal and an additive, independent noise of fixed power, is achieved when the amplitude weighting is done in proportion to the signal strength (in voltage). Thus the weighting coefficients a_i in Fig. 2 should be proportional to as good a measurement as can be made of path strengths at the corresponding delays. (In Section III-A, the same result will be obtained from a more general point of view.) The phase corrections are indicated by ϕ .

Provided that W is large enough to isolate a number of independently-fading echoes, the deleterious effects of selective fading are largely eliminated by such a

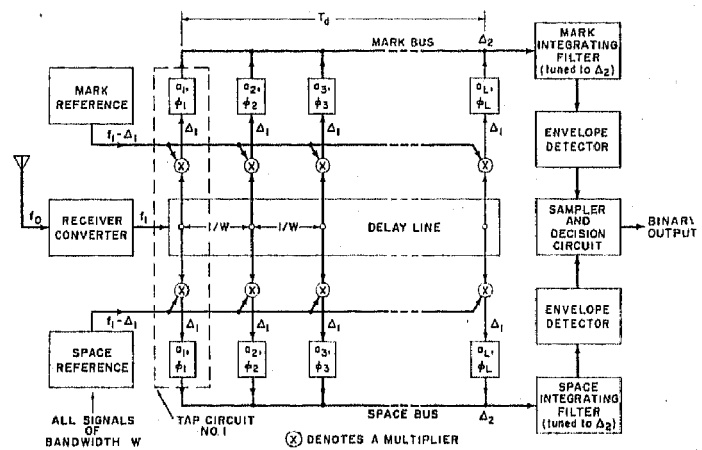


Fig. 2—Simplified block diagram of Rake receiver. The f 's and Δ 's refer to center frequencies. The a 's and ϕ 's are weightings and phase corrections, respectively based on path structure measurements.

scheme, since the path contributions are added algebraically, not vectorially. It happens that intersymbol interference is also eliminated by this Rake scheme, although a few words of explanation will be required to show that this is so. Consider the situation when the transmitter, which has been sending Mark, begins transmitting Space. Any particular echo delivers this Mark-to-Space transition to the receiver input at a different delay from the other echoes. The transition supplied by this echo, however, will be detected only at that delay line tap which corresponds to the delay of the echo, since at all other taps the echo contribution and the reference signals are uncorrelated. With the timing of the reference signals set properly so that they correlate with the last arriving echo, successively earlier echoes will correlate at points on the delay line correspondingly further delayed from its input. Thus the total propagation time from transmitter to receiver bus outputs is the time from transmitter to receiver input plus the line length to the appropriate tap, and this sum is *the same for all paths*. Hence only a single Mark-to-Space transition appears at the receiver output and intersymbol interference is eliminated.

B. Measurement, Weighting, and Phase Correction

The required weighting and phase correction is performed by the *measurement function* of the Rake receiver, which determines the path strengths and phases (including in the latter possible small variations in the positioning of the taps on the delay line). This measurement is accomplished again by the use of correlation,⁴ since, as already shown, the use of a sufficiently wide-band signal enables the various paths to be isolated. The integrating filters in this case, however, have a very long integration time, in order for the measurement to be as noise-free as possible. The integration performed

⁴ This method is practically identical to that proposed by J. B. Wiesner and Y. W. Lee [20, 21].

in the Mark and Space integrating filters is, by definition, performed over the duration of a signalling element or baud. But the time constant of each of the *measurement filters*, as they are called, should be as long as the rate of change of the multipath structure will permit. In order to assure uniform measurement regardless of the proportion of Marks and Spaces in the transmission, the measurement is provided by the sum of the Mark and Space correlations.

The method by which the correlation measurements are performed and applied is shown in Fig. 3, where a typical *tap circuit*, indicated by the dashed enclosure of Fig. 2, is depicted in greater detail. The same pair of multipliers (*A* and *B*) is used to obtain both the baud-correlations and the measurement-correlation (that is, the tones to be fed to the integrating filters and the measurement filter, respectively). The integration of the latter is performed by narrow-band filtering of the combined Mark-Space multiplier output. The amplitude of the sine wave out of this measuring filter (*C*) is proportional to the strength of the path to which the tap circuit responds, while the phase reflects the combination of the path and tap phasing. In the final multiplier tubes (*D* and *E*) this sine wave is mixed with the original outputs of *A* and *B*, thus performing the desired multiplication of these latter signals by the proper weighting function.⁵

The way in which the phase correction is applied warrants more detailed discussion. To begin with, we require that the Mark and Space references at the receiver be in the same phase relationship to each other as at the transmitter, in order that the Mark-Space modulation produce no undesirable phase discontinuities through the measuring filter. When this is done the tones of frequency Δ_1 out of the first two multipliers, *A* and *B*, are phase-coherent with each other (although only one is on at a time). Let us call this phase angle θ_1 . If θ_2 is the phase of the injection frequency Δ_2 , then the phase of the tone of frequency $(\Delta_1 - \Delta_2)$ reaching the second multiplier is $(\theta_1 - \theta_2 + \theta_0)$. θ_0 represents any additional phase shift added during filtering, and the tap circuits are aligned to have identical θ_0 . In the second multiplier the original phase θ_1 , whatever it was, cancels itself out, and only $(\theta_2 - \theta_0)$ remains. But θ_2 is the constant phase of an oscillator acting as a common injection for all taps circuits. Therefore, all such outputs at frequency Δ_2 add with the same phase angle. (Recently we have learned that a practically identical scheme of phase alignment was invented by Earp [53] for pre-detection combining of diversity receivers.)

C. Reference Signals

As we have seen, the Rake system requires Mark and Space reference signals of wide bandwidth W in order

⁵ The frequency translations of Δ_1 and Δ_2 allow mixers rather than ideal multipliers to be used for *A*, *B*, *D*, and *E*. In addition to simplifying the circuitry, the translation Δ_1 has the effect of broadening the tolerance on reference waveform synchronization [17] in elements *A* and *B*.

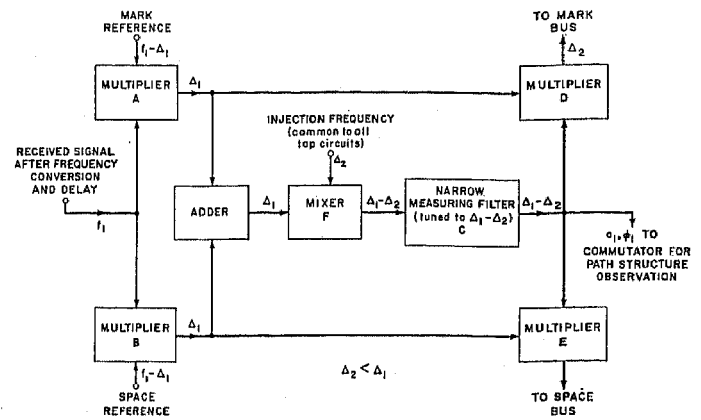


Fig. 3—Block diagram of tap circuit. (Dashed portion of Fig. 2.)

for the receiver to be able to resolve the multipath and isolate and constructively utilize the contributions of the various arriving paths. But given this requirement of wide-bandedness, what particular sort of waveforms should these signals have, out of the great number of possibilities? The choice is narrowed considerably when the following additional factors are considered:

- 1) It must be possible to store the Mark and Space signals separately at transmitter and receiver.
- 2) It must be possible to keep the timing of the reference signals at the receiver from drifting appreciably with respect to that at the transmitter, and to align the time bases to allow for the lag given by the time of flight along the longest path.
- 3) If repetitive wide-band signals are used, they must have a repeat time at least as great as the multipath duration. If the repeat time is less, paths of separation equal to a repeat time or its multiples will be indistinguishable. This condition is later interpreted in terms of sampling theory.⁶
- 4) Each signal, Mark and Space, should have an autocorrelation function as near zero as possible within the first repeat time away from the central peak (which is approximately $1/W$ wide).
- 5) The spectral components outside the assigned frequency band should be small enough not to cause interference to other services. Also, as will be seen in the next section, the spectrum should be reasonably flat inside the band.
- 6) Because high-frequency transmitters tend to be limited by peak power rather than average power, it is desired that the transmitted signal envelope be reasonably uniform, in order that a given transmitter may supply as much energy as possible to each baud.

⁶ We shall assume $S(\omega)$ to be continuous, whereas the shift-register transmission is actually a line spectrum. There is no real difficulty here, however, for by the sampling theorem [33] so long as the (complex) frequency samples are taken no farther apart than $1/T_M$, where T_M is the multipath spread, the multipath impulse response can be completely determined. The appropriate finite-interval, orthogonal interpolation functions are of the $(\sin nt/\sin t)$ variety. See p. 13 of [14].

- 7) For good discrimination in the presence of noise, Mark and Space signals should have a zero-shift cross correlation as near zero as possible. (The integration time here is the baud length T .)
- 8) Satisfactory performance of the Rake receiver measurement function requires that the signals have a long-term cross-correlation function (that is, integrated over the "ring time" of the measuring filters) that is zero for all τ shifts.

The study of *maximal-length* (or null-sequence) *binary shift register sequences* [22-24, 51] has revealed one way of constructing waveforms having the desired properties.⁷ The complete scheme for generating Mark and Space signals, which is duplicated at transmitter and receiver, is shown in Fig. 4.

The repetitive sequence is generated by a self-driven binary shift register in which the modulo-two sum of the digits appearing in the output and certain of the intermediate stages is fed back to the input. For an n -stage shift register, the maximal-length binary sequence has a period $m = 2^n - 1$, and is generated through suitable choice of which intermediate stages are fed back to the input. One such shift register can easily be set up at the transmitter, and another at the receiver (requirement 1). Each can be driven by pulses derived from crystal oscillators (requirement 2) with a period m made as long as necessary by choosing a sufficiently large number of stages n (requirement 3).

Maximal-length shift register sequences were chosen in preference to other classes of binary sequences because of a peculiar property that is useful in satisfying requirement 4: if we let positive and negative impulses represent the binary variable, the shift-register output is, of course, a sequence of such pulses repeating every m pulse. The autocorrelation function of this maximal-length sequence is a sequence of impulses, each of area (-1) except at the origin and integral multiples of the period m , where there are impulses of area m . This autocorrelation function is, for large m , very nearly that of a repeated impulse, and hence the power density spectrum exhibits a nearly uniform comb of spectral lines.⁸

To limit the frequency spectrum (requirement 5) the shift register output sequence is passed through a rectangular, band-pass filter of width W cycles per second. The envelope of the resulting signal fluctuates considerably, contrary to requirement 6, but by following the filter with a sharply limiting device this fluctuation can be suppressed. Such a nonlinear operation of course spreads the spectrum somewhat (resulting, for example, in a spilling of 10 per cent of the power outside the original band when the output pulse rate of the shift

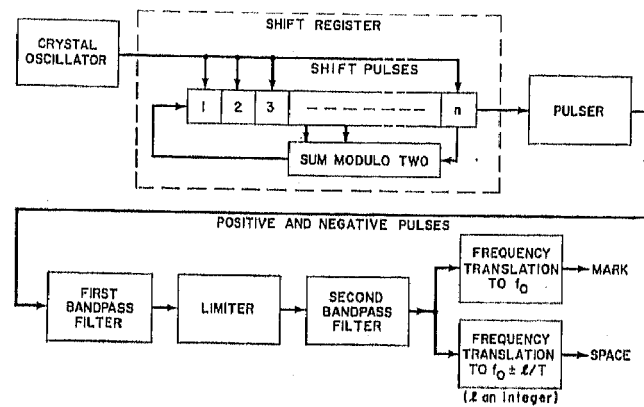


Fig. 4—A method of generating suitable reference signals.

register is large enough compared to W to yield approximately Gaussian noise at the filter output [25]). Accordingly, a second filter, identical to the first, follows the limiter to restore the nearly rectangular power spectrum without producing as severe envelope fluctuations as are present in the output of the first filter.

To satisfy requirement 7, we seek orthogonal, wide-band Mark and Space waveforms having about the same energy. If one reference signal, say Mark, has only slight envelope fluctuation (as described in the preceding paragraph), a frequency shift of only (l/T) is adequate to produce another nearly-orthogonal waveform that we can use for Space, even though the Mark and Space spectra may overlap considerably. Here l is any nonzero integer and T is the baud length.⁹ This frequency shift is the final operation shown in Fig. 4. The long-term zero cross-correlation function stipulated by requirement 8 can be attained only if the long-term power spectra of the Mark and Space signals are disjoint. This will be assured so long as the frequency shift is not equal to or near a multiple of the reciprocal of the shift-register repeat time.

D. Integration and Decision

The integrations upon which each Mark-Space decision are based are required to be performed over the corresponding baud interval of length T . A filter of very high Q (time constant several times T), tuned to the bus frequency Δ_2 , will faithfully perform the integration of the tap circuit outputs. It must be "quenched" or "dumped," however, after each decision to prevent ringing from the previous baud from carrying over into the succeeding baud intervals [8, 26].

Just previous to quenching, samples are taken of the

⁹ Proof: Let the Mark baud-waveform be represented as $R(t) \cos [\omega_0 t + \phi(t)]$; the frequency-shifted (Space) waveform is then $R(t) \cos [(\omega_0 + 2\pi l/T)t + \phi(t)]$. Neglecting terms near $2\omega_0 t$, the cross-correlation envelope at the time of sampling is

$$\max_{\theta} \int_0^T R^2(t) \cos (2\pi l t/T + \theta) dt \approx 0$$

for $R(t)$ approximately constant, and tends more toward zero as l increases. See also [8].

⁷ The use of short pulses would fit naturally into the Rake concept, if it were not for the limitation of requirement 6.

⁸ The effect of the small negative impulses between the large positive ones in the autocorrelation function is to suppress almost completely every m th tooth of the frequency comb, beginning at zero frequency.

envelopes¹⁰ of both integrating filter outputs, and decision is based on whether the Mark or Space sample is the larger. The timing required to make the sampling and quenching operations occur exactly at the end of each received baud implies knowledge of the transmitter modulation timing. This is readily available, because of the degree of the transmitter-receiver synchronization already required of the reference sources.

III. THEORETICAL FOUNDATIONS

In this section we present the communication-theoretical arguments that led to the Rake receiver design. Two rather distinct mathematical treatments have been pursued, which between them encompass a fairly realistic and complete set of assumptions. As yet, no unified theory has been forthcoming which includes the entire set of assumptions.

Both analyses are based on statistical decision theory [27, 28] and both lead, with some intuitive extension, to the Rake receiver previously described. We shall first discuss the simpler and more physical argument, and later briefly touch upon the more abstract analysis that actually first suggested the Rake configuration.

In the application of statistical decision theory, it is customarily assumed that the transmitter, its signals, and the channel are specified, at least on a statistical basis. Here, we shall assume that the receiver has knowledge of the possible transmitted waveforms and their *a priori* probabilities, and complete statistical knowledge of all random elements in the transmitter and channel. We seek the best such receiver. Generally the "best" receiver is taken to be that which achieves the minimum average "cost," suitably defined, of wrong decisions made by the receiver [27]. In the communication problem under study it appears that the appropriate cost function is simply the probability of error—that is, a Mark being mistaken for a Space is no worse nor better than the inverse error. This assumption becomes even more reasonable when we add the additional condition that the *a priori* probabilities of transmitting a Mark or a Space are to be equal. It has been shown that the corresponding optimum receiver, called by Siegert the Ideal Observer [29] is one that, from the received waveform, computes the *a posteriori* probabilities that a Mark, or a Space, was transmitted, and decides in favor of the symbol having the larger probability.

A. Analysis for Multipath Assumed to Be Perfectly Measurable

Using this philosophy, it is straightforward to find the optimum receiver in the case of a channel perturbed

¹⁰ The phases of the signals on the output buses are fixed and known, so that in principle synchronous (coherent) detection could be employed to some advantage [4, 47]. In terms of probability of baud decision error, however, the equivalent transmitter power advantage of synchronous over envelope detection is only 1.14 db at an error rate of about one teletype character in three lines of copy, and becomes even less at smaller error probabilities. See [18], (13) and (39) with $\lambda=0$ (orthogonal signals).

solely by additive white Gaussian (thermal or shot-effect) noise [30, 31]. Assuming that the Mark and Space waveforms have equal energy, the Ideal receiver in this case simply cross correlates the received waveform with the two stored references, and bases its decision on the symbol yielding the larger correlation. (Thus the simple correlation receiving system of Fig. 1, used previously for explanatory purposes, is seen to be an optimum system against white noise under the foregoing assumptions.)

The introduction of an arbitrary linear or nonlinear filter into the channel, preceding the noise, can be accommodated by a simple extension of this result, provided that the receiver knows or can measure the filter characteristic exactly. It is apparent that the optimum receiver in this case first passes its local reference waveforms through filters identical to the one in the channel and then performs cross correlations as before. (We assume that there is a negligible difference in the energies of the filtered reference wave forms.)

The above requirement that the receiver have complete knowledge of the channel filter is rather unrealistic for an ionospheric channel, since exact measurement of such a randomly-varying filter characteristic cannot be made so long as any noise is present. Fortunately, the multipath "filter" usually changes fairly slowly, and, furthermore, can be considered linear, so that good measurement accuracy can be achieved by a number of procedures, such as the cross-correlation method outlined previously. It is then plausible to employ the results of the measurement in constructing filters to process the stored waveforms for correlation, as just stated, ignoring the small measurement errors.¹¹

We now examine more carefully requirements on the cross-correlation method of measuring the multipath characteristic described in Section II-B, and the process of applying it to correct the stored waveforms. We assume transmitted signal $x(t)$ to have a band-limited spectrum $S(\omega)$, and the multipath to be varying so slowly relative to the transmission bandwidth that a quasi-stationary analysis is allowable. (The usual Fourier-transform relations are assumed to hold between correlation functions and power spectra on a finite-observation-interval basis.) Then in order to measure everything about $H(\omega)$ (the "instantaneous" transfer function of the multipath) that is relevant, it is only necessary, and only possible, for the receiver to measure that portion $H_M'(\omega)$ of $H_M(\omega)$ occupied by the transmission.⁶

$$H_M'(\omega) = \begin{cases} H_M(\omega); & \omega \text{ in transmission band} \\ 0 & ; \text{ elsewhere.} \end{cases} \quad (1)$$

$H_M'(\omega)$ may be found from the cross-correlation function $\phi_{xw}(\tau) = x(t)w(t+\tau)$ of the transmitted signal $x(t)$

¹¹ Such a procedure closely parallels a suggestion of W. L. Root and T. S. Pitcher [32]. See also [14], pp. 12-14, where the use of a frequency-group transmission like that provided by the shift-register is suggested to accomplish the measurement necessary to correct the reference waveforms.

and the received signal $w(t)$.¹² (It is assumed that $\phi_{zw}(\tau)$ is measured with an effective integration time appropriate to the rate of change of $H_M'(\omega)$.) Although $x(t)$ is of course not available to the receiver, $\phi_{zw}(\tau)$ can be found from correlating the received signal with the sum of both stored waveforms, providing their long-term cross-correlation function is zero for all τ shifts. Now the Fourier transform of this $\phi_{zw}(\tau)$ is the cross-spectral power density spectrum, $\Phi_{zw}(\omega)$ which is equal to $S(\omega)H_M(\omega)$ [35]. So,

$$H_M'(\omega) = \begin{cases} \Phi_{zw}(\omega)/S(\omega); & \omega \text{ in transmission band} \\ 0 & ; \text{ elsewhere.} \end{cases} \quad (2)$$

If $S(\omega)$ is now assumed to be constant within the band, (2) says that the impulse response $h_M'(t)$ of the receiver filters that should correct each stored symbol is given directly by

$$h_M'(t) = K\phi_{zw}(t) \quad (3)$$

where K is a constant. Finally, since $h_M'(t)$ has a spectrum limited to the transmission bandwidth W , it is only necessary to measure $\phi_{zw}(\tau)$ at points $1/2W$ apart in τ , or, since $S(\omega)$ is a bandpass spectrum, to find the envelope and phase of $\phi_{zw}(\tau)$ at points $1/W$ apart [33]. Even when the effect of channel noise is included in the measurement, any closer spacing of the sampling points gains nothing, since the error in measuring $\phi_{zw}(\tau)$ for a particular received $w(t)$ is, as a function of τ , also limited to bandwidth W .

The complete band-limited impulse response $h_M'(t)$ may be found by interpolating from its sample values, using interpolation functions of the form $(\sin t/t)$. Likewise, the stored symbols which are to be passed through $h_M'(t)$ can be expressed as a series of samples with interpolations of the same form. By virtue of the orthogonality of these interpolation functions, it is easy to show that the convolution resulting from filtering the stored waveforms with $h_M'(t)$ is exactly equivalent to multiplying the sampled values of the stored signal with those of $h_M'(t)$ and summing the products. Hence both the measurement and correction operations can be performed on a discrete basis, using multiple-tap delay lines with taps $1/W$ apart.

A block diagram of this Ideal receiver is shown in Fig. 5. Difference-frequency correlation² is employed to obtain the phase at the sampling taps. The sampled values of $\phi_{zw}(\tau)$ are obtained directly from the tap outputs they multiply, in order to keep the local filtering in alignment with the channel multipath filter. As mentioned earlier, both stored waveforms contribute equally to the measurement of $\phi_{zw}(\tau)$.

To show that this result is equivalent to the receiver described in Section II, we first rearrange the multiplying and adding elements of Fig. 5, as we may do freely

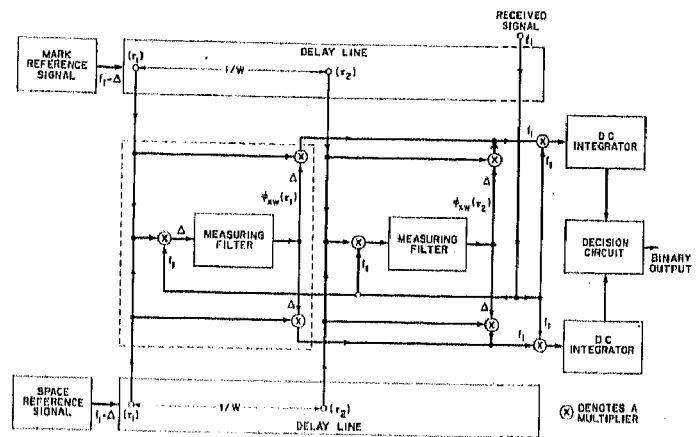


Fig. 5—Diagram of Ideal, delayed-reference receiver. The f 's and Δ 's appear as in Fig. 2. The $\phi_{zw}(\tau_i)$ are the multipath measurements. Measuring filters are tuned to Δ and have bandwidth approximately equal to the multipath fluctuation rate.

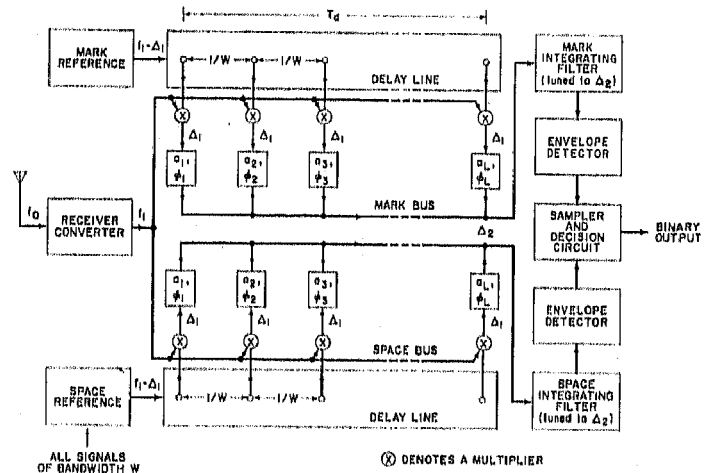


Fig. 6—Simplified block diagram of Ideal, delayed-reference receiver. This is Fig. 5 rearranged, employing the terminology of Fig. 2.

because of their linearity. We arrive at the configuration shown in Fig. 6 where the terminology is the same as that of Fig. 2. The double-difference frequency scheme of Fig. 2 has also been introduced, but results in no basic change in the operation. Envelope detectors are now indicated as well, but are incidental. In arriving at the final equivalence between Fig. 6 and Fig. 2, we study the signals and noise on the Mark and Space buses. At this point the only difference is whether the delay line appears in the incoming signal or in each reference signal. We will term the schemes of Fig. 2 and Fig. 6 the *delayed-signal* and *delayed-reference* configurations, respectively.

Finally, it is necessary to show that, independent of the form of the reference signals employed, the output snr from the integrating filters is substantially the same for both configurations, under the assumption that the length of the delay line T_d is significantly smaller than the baud length T .¹³ Each integrating filter responds to

¹² G. L. Turin has shown that under certain conditions cross correlation is the optimum measuring operation for white noise present in the channel. See [34].

¹³ We are indebted to Prof. R. M. Fano for the interpretation given in the present paragraph.

signals only within about $\pm 1/T$ of the frequency Δ_2 . Therefore, the noises adding on the common buses can be considered sinusoids of frequency Δ_2 having a fluctuation period no shorter than T , regardless of the form of reference signal. The only difference between the tap circuit contributions of the delayed-signal scheme (Fig. 2) and those of the delayed-reference scheme (Fig. 6) is that the latter are staggered in time by various fractions of T_d , and since such staggering is therefore small compared to the significant fluctuation period of the contributions, we conclude that the noise outputs of the two configurations are equivalent.¹⁴

There are three practical advantages of the delayed-signal scheme over the delayed-reference scheme. First, one delay line instead of two is required. Second, in the latter configuration, corresponding taps in the Mark and Space lines would have to be adjusted to and kept in phase coincidence. Third, coherent intersymbol interference (eliminated in the delayed-signal scheme) is still present in the latter scheme, as can be verified by repeating the same line of reasoning given in Section II-A.

In this analysis we have ignored the consequences of imperfect measurement of the path structure. The question of what form the best receiver takes when the path fluctuations become rapid cannot be answered by this simple reasoning. The next part summarizes the theoretical study that originally led to the Rake configuration. This analysis allows the paths to fluctuate, without assuming at the outset an explicit measurement function in the receiver.

B. Analysis for the Assumption of a Discrete Path Structure with Known Time Delays [42]

In order to extend the analysis to include random variations in the multipath structure, we must at present make the restriction that there are a finite number of discrete paths whose time delays are known *a priori* to the receiver.¹⁵ The assumptions with which Section

¹⁴ There is an aspect of the configuration of Fig. 2 that is related to the familiar notion of the *matched filter* (or conjugate filter) [15, 36]. With a simple rearrangement of the multipliers and adders, the delay line can be seen to be part of a filter matched to the multipath characteristic; that is, the filter has an impulse response that is the time reverse of the path structure. The output of this filter would then be fed directly to a single pair of Mark and Space correlators. This illustrates the fundamental difference between the Rake receiver and schemes which seek to pass the received signal through a filter that is the *inverse*, rather than the *conjugate*, of the multipath transfer function.

¹⁵ In an important related study by Turin [37, 38], a result is obtained for the situation in which the strengths and delays of a number of discrete paths are presumed unknown to the receiver *a priori* [38], p. 160. This result is derived from a formula [38], (24) which can yield the Rake receiver in a manner similar to the approach of Section III-A.

Both Turin's study and ours [42] assume knowledge of certain channel parameters which must ultimately be measured in some manner. However the concept of symbol correction using a cross-correlation method of "instantaneous" path measurement, as employed in the Rake receiver, materializes as a unit (with some intuitive modifications mentioned in the text shortly) from the analysis of [42]. Turin, Root, and Pitcher [32], and Section III-A of this paper, on the other hand, logically unite the rather distinct operations of path measurement by correlation, and symbol correction using path measurement.

III began apply here, as well as the condition that the additive interference be white Gaussian noise. The paths are taken to be of the Booker-Gordon "scatter" type [39, 40], and to vary independently of each other, the transmitted signal, and the noise.¹⁶ It is further assumed that the complete statistical description of each path is available to the receiver, in the form of $\phi_p(\tau)$. The quantity $\phi_p(\tau)$ is the "correlation function of the p th path," placing in evidence its average strength and rate of variation.

For this model, the corresponding Ideal receiver for the detection of isolated bauds has been deduced [42]. The exact solution, in open form, is given in (20) through (26) of [42]. (Unless otherwise stated, all numbers in this paragraph refer to [42].) An approximate closed-form solution appears in (49), and an analog computer for this equation is shown which bears a near-equivalence to the Rake configuration, having a tapped delay line, multiplying and integrating elements, and filters. The filters are matched in bandwidth to corresponding path stabilities, and may be viewed as performing the measurement-correction function of Section III-A in the present paper. The outputs to the decision circuit are designated by $\log_e L_k$; $k=1$ for Mark, 2 for Space. A bias block shown in the computer, and given by the last term in (49), may be presumed the same for Mark as Space, and hence eliminated from the decision.

By lumping together the Mark and Space measuring filters, leaving the resulting filters unquenched after each baud decision, and finally by placing the delay line taps at intervals of $1/W$ apart, to capture all paths that may arise,¹⁷ the configuration of Fig. 5 in the present paper is reached. The final Rake receiver of Fig. 2 then follows in the same manner as in Section III-A.

IV. EXPERIMENTAL REALIZATION OF RAKE SYSTEM

In this section we shall describe the construction of an experimental prototype Rake system and mention some of the problems encountered in its operation. Since a large portion of the circuitry is quite conventional, considerable attention will be devoted only to those elements of the system that may be novel in realization or application.

¹⁶ We neglect the possible presence of a specular component which may be encountered, in addition to "scatter" (random component) in ionospheric propagation below the maximum usable frequency (muf) [41]. It has been shown by Turin [38] that at small signal-to-noise ratios such specular components play a dominant role in conveying information. From observations during recent experiments, however [43], we do not feel that conclusive knowledge is yet available of the relative amplitudes of the specular and scatter components to be expected in below-muf propagation. This question is being studied further by M. Balser and others in Lincoln Lab., Mass. Inst. Tech., Lexington Mass.

¹⁷ Turin's work ([37], pp. 40-44) indicates that when the path time delays are unknown *a priori*, the weighting of the tap contributions (Section II-B) to the output buses of Fig. 5 should be a non-linear function of the measurement. A very rough approximation to this refinement is considered in Section IV-F.

Fig. 7
m
rig
en

A. T

Th

stage

from

maxi

The s

occur

sequ

longe

The p

sec o

a 10-l

filter

serve

trum

Fig. 8

is spru

the sp

a seco

the sp

sultin

and 8

sidera

Fre

venier

and th

is a to

filter

155 k

for M

integr

18 M

from 30

(Wave A

X10³)

cent.

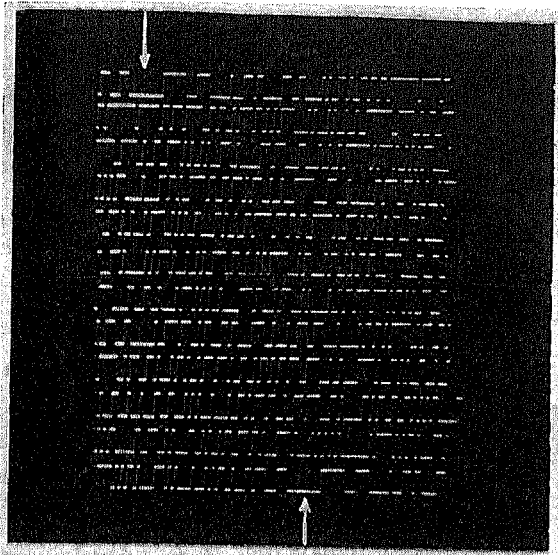


Fig. 7—Maximal-length shift-register sequence. $n=10$ stages, period $m=1023$. The entire sequence is shown, reading from left to right and top to bottom, with some overlap. The beginning and end are shown by arrows.

A. Transmitter

The transmitter shift-register (Fig. 4) has $n=10$ stages, and the binary sum of the last stage and fourth from last stage is fed back to the input stage to yield a maximal-length sequence of period $m=2^{10}-1=1023$. The sequence is shown in Fig. 7. The driving clock pulses occur at a 120-kc rate, so that the repeat time of the sequence is $1023/(120 \times 10^3) = 8.525$ msec, which is longer than normal ionospheric multipath duration.¹⁸ The pulser passes positive and negative pulses of 0.1 μ sec duration into a four-pole Tchebycheff filter having a 10-kc half-power bandwidth, centered on 455 kc. The filter output is shown in Fig. 8(a), next page, and is observed to have large envelope fluctuations. The spectrum of the output of this first filter is presented in Fig. 8(b). Following the band-pass limiter the spectrum is spread, as shown in Fig. 8(d), and in order to suppress the spectral "tails" the limited signal is passed through a second Tchebycheff filter, 10 kc wide and centered on the spectrum. The waveform and spectrum of the resulting final wide-band "carrier" are shown in Fig. 8(e) and 8(f), respectively. The envelope is seen to be considerably more uniform than that of Fig. 8(a).

Frequency-shift modulation of the "carrier" is conveniently accomplished between the band-pass limiter and the second filter. The injection for this conversion is a tone of average frequency 155 kc, so that the second filter is actually centered on 300 kc. The deviation from 155 kc is $\pm 2/(22 \times 10^{-3}) \approx \pm 90.90$ cps (arbitrarily + for Mark, - for Space), corresponding to an exact integral multiple of the reciprocal of the 22-msec baud

length, as required for orthogonality. (The output of the conventional, motor-driven teletype distributor is here electronically retimed to provide bauds of this length, with accurately determined end points [26].)

The 300-kc signal from the second filter is heterodyned to the allocated transmission frequency. Harmonics of a 100-kc Western Electric 0-76/*U* crystal clock, from which the 120-kc shift-register driving pulses and the baud-timing are also derived, are employed to perform the major portion of the frequency translation. Other crystal oscillators accomplish the remaining interpolation.

B. Receiver Reference Sources

The Mark and Space reference sources at the receiver are identical to those at the transmitter, except that there is a fixed downward translation of frequency by about 20 kc just ahead of the second filters which are now centered on 435 kc. (In Fig. 2 and Fig. 3, $f_1=455$ kc, $\Delta_1=20$ kc, $\Delta_2=9$ kc.) The Mark-reference injection frequency is (20 kc + 90.90 cps) and the Space-reference injection is (20 kc - 90.90 cps). The phase of the 181.80 cps Mark-Space deviation at the receiver (*i.e.*, the tone that would be obtained if Mark and Space references were multiplied together) must be kept in good alignment with the transmitter deviation. This is required so that modulation produces no phase transients in the receiver measuring filters, as explained in Section II-B. The required high order of deviation-frequency agreement and stability is obtained by first providing crystal oscillators at (155 kc - 90.90 cps) and (20 kc - 90.90 cps) for the Space frequencies at the transmitter and receiver, respectively. For Mark, the 181.80 cps $\approx 4/(22 \times 10^{-3})$ deviation from these frequencies is obtained at both transmitter and receiver from a harmonic of the 22 msec baud timing using single-sideband modulation. Once the phasing of the transmitter and receiver 181.80 cps deviations is brought into agreement, the precision of the 100-kc clocks from which the baud timing is derived is sufficient to maintain it for many hours.

C. Conversion and Delay of Received Signal

The rf signal is received and translated to the standard intermediate frequency of 455 kc by a conventional receiver (R390/URR). Double conversion is employed, the major portion of the frequency translation being performed by the harmonics of the receiver 100-kc crystal clock. As at the transmitter, other crystal oscillators supply the remaining interpolation.

The IF signal is fed into a cascade of two ultrasonic delay lines, of a novel type,¹⁹ each 1.5 msec long and having about 50-kc bandwidth at 455-kc center frequency. This multipath of up to 3 msec duration can be accommodated. One of the lines is shown in Fig. 9.

¹⁸ Measurements of the spectrum of this sequence in the region from 30 cps to 10 kc were made using a General Radio Model 736-A Wave Analyzer. The amplitudes of the spectral lines, spaced $1/(8.525 \times 10^3) \approx 117.30$ cps apart, were found to be uniform within ± 1 per cent.

¹⁹ These lines were designed and built under the guidance and with the invaluable assistance of R. M. Lerner, H. Penfield and L. P. Romano, of Lincoln Lab., M.I.T., Lexington, Mass.

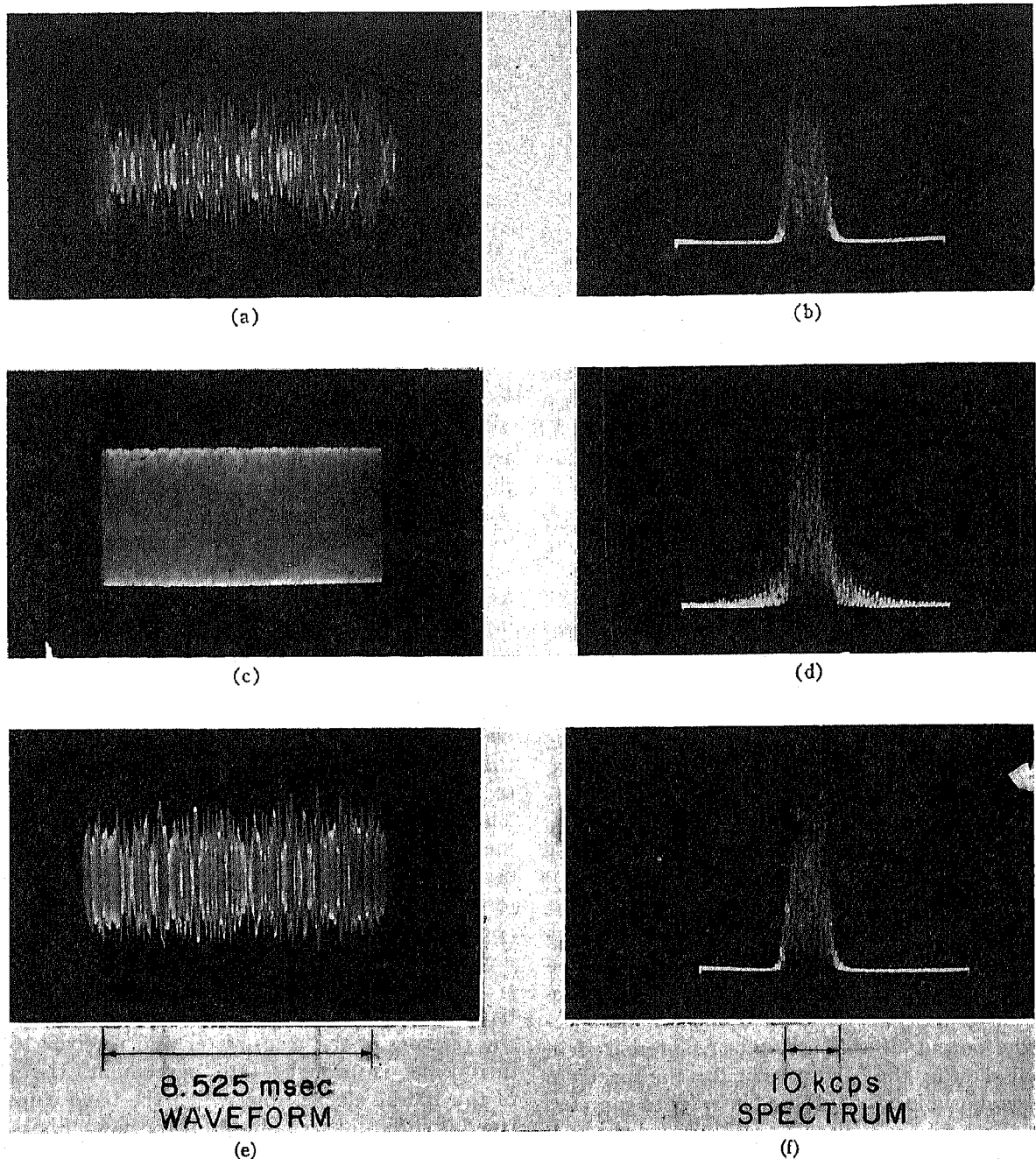


Fig. 8—Reference source waveforms and spectra. (a) and (b) First filter output. (c) and (d) Limiter output. (e) and (f) Second filter output.

D. Tap Circuit

The following parameters of the tap-circuit block diagram of Fig. 3 have been established for the experimental Rake system. From the preceding remarks, the center frequency f_1 of the received signal, after conversion and delay, is 455 kc. The reference sources are centered at 435 kc, so that Δ_1 is 20 kc.²⁰ The reference

²⁰ The 455 and 435-kc figures just mentioned refer to Mark and Space signals alike. However, the difference between Mark and Space is a 181.80-cps shift of the internal structure within the nominal band. Therefore if Mark is received, for example, the Mark multiplier A produces a $\Delta_1=20$ -kc tone, but Space multiplier B produces a tone 181.80 cps away from Δ_1 . The latter is suppressed in the Space integration (see next section).

frequency (and therefore the output bus frequency) Δ_2 has been chosen to be 9 kc, after consideration of the strengths and locations of various intermodulation products and the ease of suppressing them by series or parallel traps. The measuring filter is therefore centered on $\Delta_1-\Delta_2=11$ kc, and its bandwidth is made 1 cps to correspond roughly to the average rate of ionospheric fluctuations.

The multipliers and mixer are single type 6AS6 tubes, which have already been widely used as analog multipliers. Operated on a difference-frequency basis with proper biases on the control and suppressor grids, they are very linear over a dynamic range (at their output) of

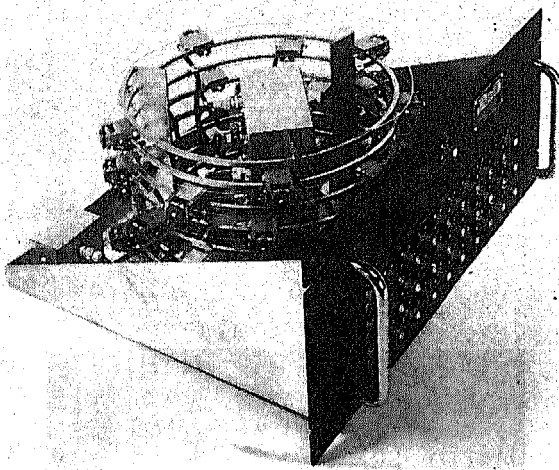


Fig. 9—Delay line. 1500- μ sec helical ultrasonic line with the driving circuits in the center, driving transducer at back left, and termination at top right. The center frequency is 455 kc, bandwidth 50 kc; the injection gain input-first tap 10 db, insertion loss of wire element 28 db, ratio of signal to spurious, 25 db. Line uses 1/32-inch diameter invar rod, driven piezoelectrically in longitudinal mode through exponentially tapered matching section. Driving transducer is quarter-wavelength cylinder of barium titanate. Low- Q output coils are biased by permanent magnets. Rod is terminated in winding of tape clamped in rubber (not shown).

100 db. The gains in the tap circuit are set so that as the received signal grows large the grids of the second multipliers reach saturation at approximately the same point. The measuring filter is an 11-kc NT-cut, flexure-mode bar crystal, driven in the series-resonant mode from a cathode follower. The complete assembly of the 30 plug-in tap circuits and two delay lines, which comprises the major part of the Rake receiver, is shown in Fig. 10.

E. Integrating Filters and Decision Circuitry

The integrating filters have high- Q coils enclosed in temperature-stabilized ovens. They are tuned to 9 kc and employ Q -multiplication circuits for good integration linearity. The quenching is accomplished by diodes and is completed in a few hundred microseconds.²¹

Envelope detectors, connected in series opposition, are applied to the two integrating filter outputs, and the net dc voltage is applied to the decision circuit, which consists of a limiting dc amplifier, sampler, and Schmitt trigger driving the teletype printer.

F. Auxiliary Features and Devices

Effective operation of the Rake system with a minimum of manual intervention calls for the use of automatic frequency control (afc), automatic gain control

²¹ Integration patterns resemble those shown in Fig. 3 of [8]. For example, if the output of the Mark filter is observed, a linear buildup is seen when a Mark is transmitted, while a sequence of four small scallops results when Space is transmitted. The latter waveform corresponds to the frequency separation of four reciprocal-baud-lengths between Mark and Space (see Section IV-A).

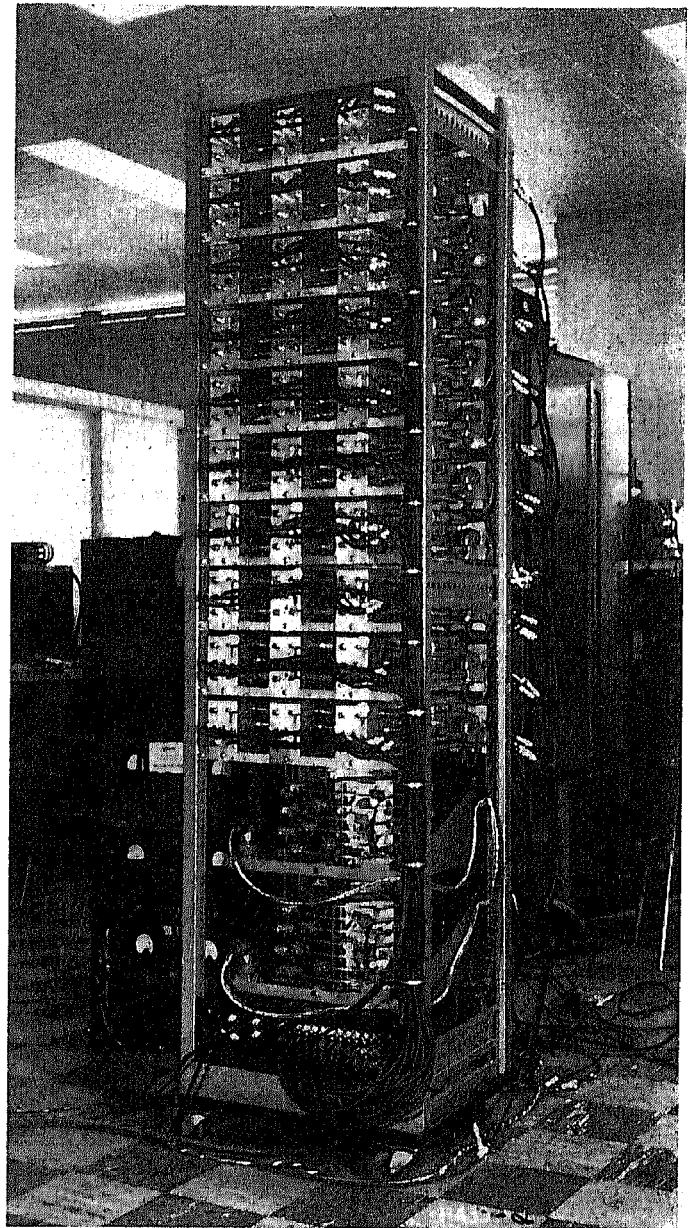


Fig. 10—Rake receiver rack. Inputs are through cables in rear; output buses run across front. Two delay lines and commutator located at bottom.

(agc), and a means for observing the multipath structure. These auxiliary devices will now be discussed.

For satisfactory operation, over-all frequency alignment of the system must be kept to within a fraction of the measuring-filter bandwidth, regardless of drift in the transmitter and receiver converters and ionospheric Doppler shift. The frequency-sensitive element for the required afc can be realized by comparing the phase of the sum of the two Rake bus outputs with that of the injected frequency Δ_2 . The sharp phase characteristic of the measurement filters thus provides a sensitive indication of frequency alignment. If it should happen that different paths have widely different Doppler shifts, this scheme may have to be modified considerably.

Changes in the received signal level appear as level changes on the Mark and Space output buses in a square-law relationship because of the weighting. Since the integrating filters will not accommodate a very great dynamic range, agc must be employed so that such level fluctuations are not great.

Three additional manual adjustments are needed from time to time. First, care must be taken that the entire multipath structure is contained within the multiple-correlation interval T_a ; this is accomplished by adding or subtracting occasional shifting pulses from the receiver reference shift register. Second, when only a few tap circuits are responding to significant paths (for example, when there is only one ionospheric path), it is well to disconnect the remainder from the buses. This eliminates the small amount of noise these circuits contribute due to noise fluctuations in their measuring filter outputs (which ideally should be zero when no path is present but are not because of the nonzero measurement filter bandwidth).¹⁷ Third, the bandwidth of the measuring filters should be kept in correspondence to the path stability, although this is a far-from-critical adjustment.

These three manual adjustments made during operation, together with such initial adjustments as are required when the system first goes "on the air," are facilitated by an auxiliary feature of the Rake receiver. The multipath measurement is presented continuously on an oscilloscope by means of a commutator which samples the measuring filter outputs, as indicated in Fig. 3. The operator can thereby study the ionospheric behavior at all times, simultaneously with the reception of intelligence. Since dynamic path phase information is also contained in the measuring filter output, it is possible to observe multipath phase stability over long distances, by employing a suitable horizontal sweep on the oscilloscope. Similar observations have already been reported elsewhere [43].

V. TEST RESULTS

On-the-air tests of the Rake system over a transcontinental circuit were conducted during the summer of 1956. The transmitter was situated at the U. S. Army Radio Station AAG in Davis, Calif., where a maximum undistorted power of 20 kw was available from a Collins FRT-22 single-sidebanded transmitter. The receiver was located at the Army receiving facility in Deal, N. J. Both transmitter and receiver employed rhombic antennas of about 12-db gain. Transmissions were made near 8, 12, and 17 mc at various times of day.

Fig. 11 shows some typical path structures observed by commutating an oscilloscope along the measurement outputs of the successive Rake receiver tap circuits. Fig. 11(a) represents the usual situation in which discrete paths are present, while Fig. 11(b), taken immediately after a sudden ionospheric disturbance, presents the rarer case of an apparent continuum of paths. It was

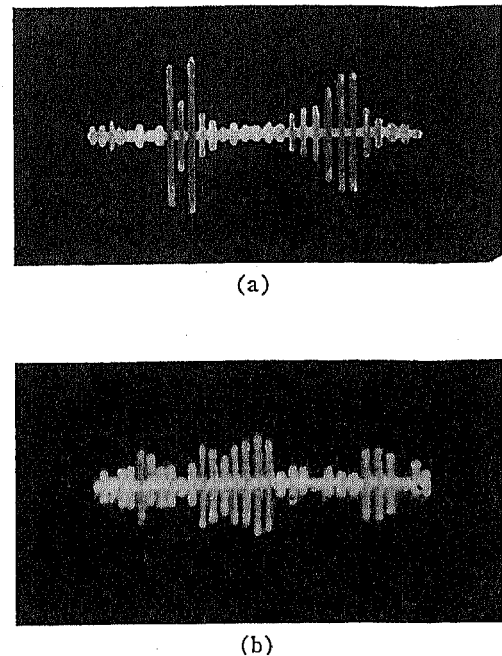


Fig. 11—Path structures observed on Rake receiver. Both observations made in 12-mc band with 3 milliseconds observation interval. (a) Fine structure in left group and broadness of right group indicate two closely spaced pairs of paths, with interpath separation of 1.4 milliseconds. (b) Path continuum immediately after a sudden ionospheric disturbance.

possible to observe a long continuum of paths of up to 5 msec duration by operating the system in the region of high absorption below the so-called lowest usable frequency (luf).

Fig. 12 presents an observation of the selective fading of the wide-band signal, as seen on a spectrum analyzer. On one occasion, a two-path structure was encountered near 17 mc in which there appeared to be a differential Doppler shift between the two paths equal to 1.5 cps. This was measured by tuning the highly stable injection to the Rake receiver (R390/URR) for maximum response to first one path and then the other, and confirmed by observing on the spectrum analyzer the rate at which the selective fades drifted across the transmission band. (See also Fig. 8 of [9].)

Satisfactory performance of the system was obtained for a wide variety of multipath structures. On one occasion 1800 lines of teletype copy (representing nearly six hours of operation) were printed at the receiver without error. Although experimental evidence is not yet conclusive, it appears that the Rake system performs best at frequencies well below the maximum usable frequency (muf) where there is enough multipath spread that there is small probability of a simultaneous fade on all paths. Once, when the multipath extended over 5 msec, satisfactory printing was obtained with a Rake observation interval of only 1.5 msec. This indicates that it is not absolutely necessary to make use of all significant paths, although performance against noise naturally improves as more paths are included.

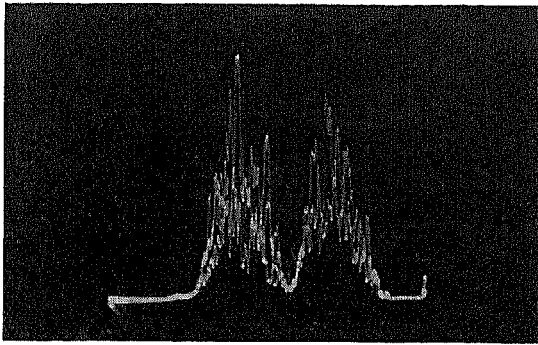


Fig. 12—Selective fading as observed on a Panadaptor. The individual frequency components are not resolvable. The width of the spectrum is 10 kc.

Very rapid (≈ 10 – 20 cps) fluctuations of the multipath structure were occasionally encountered during disturbed conditions, and this resulted in poor system performance, chiefly because of the inability of the measurement filters (whose bandwidth had not been made adjustable) to respond rapidly enough.

A brief test was made which confirmed that a delay-line tap spacing of $100 \mu\text{sec}$ was as efficient as $50 \mu\text{sec}$, and that a spacing of $150 \mu\text{sec}$ resulted in marked degradation in performance, in accordance with the sampling analysis of Section III-A.

It was of interest to compare the performance of the Rake system with that of an fsk system operating under the same conditions. A standard frequency shift of 850 cps was employed, and fsk receivers of both the conventional (CV-116/URR) and predicted-wave [26] type were employed. The fsk receivers were operated both with and without dual space-diversity, the diversity combining being accomplished through a common limiter in the CV-116, and through diode combining in the predicted-wave receiver.

Comparisons between the Rake and fsk systems were made on the basis of probability of teletype character error (five bauds make up a character) during times when the only additive channel disturbance was additive noise. The transmitter was calibrated to establish equal average power levels for the two systems, and alternate transmissions of groups of ten lines each (lasting about two minutes) were made using Rake and fsk at various levels. Times when the only additive channel interference was solely noise were rare, and consequently only a small amount of comparison data was taken. Two error runs are presented in Fig. 13. In Fig. 13(a) comparison is made with the conventional CV-116 fsk receiver, while in Fig. 13(b) predicted-wave reception is included as well. (It should be noted that comparison at these high error rates cannot be extrapolated to the error rates of commercial interest.) Both these runs employed space-diversity reception with the fsk transmission, but with Rake space diversity is not required and was therefore not used.

Experiments were made to determine the extent to which the information rate could be sped up by decreas-

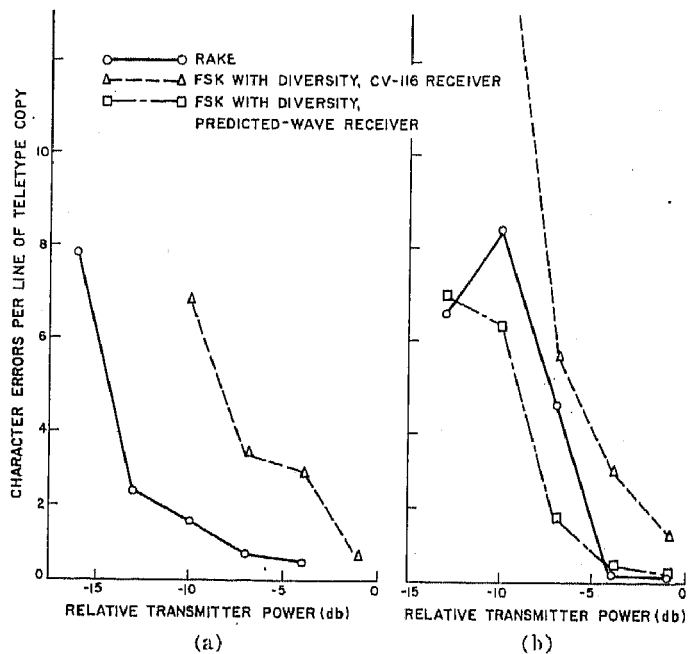


Fig. 13—Comparisons of Rake vs fsk. (a) Observed 2100 EST, 12-mc band. (b) Observed 0100 EST, 8-mc band.

ing the baud length. This was an attempt to take advantage of the absence of coherent intersymbol interference in the Rake receiver. It was found that the decreased integration time (baud length) caused a prohibitively high error rate (≥ 1 error per line) for values of $T \leq 5$ msec (or $TW \leq 50$).

VI. CONCLUSION

The preceding pages have outlined the features of a new technique for dealing with a channel perturbed by random multipath disturbances and additive noise. This Rake system derives its name from the operations performed at the receiver, where a number of correlation detectors are provided at uniform delay increments, with their outputs added. The addition takes place only after each correlated output has been suitably weighted and phase-corrected for maximization of the snr of the sum. By employing wide-band signals, such a procedure allows algebraic rather than vectorial addition of the signals from the various paths, thus eliminating the degradation usually associated with selective fading. This method of detection, weighting, and combining was originally suggested by the study of optimum probability-computing receivers for multipath and noise (Section III-B).

It was pointed out that the original concept, developed in the communication-theoretical derivations of Section III, can be modified by inserting the required delays in the incoming signal rather than the reference signal. The result of this alteration is that a reconstitution or collapsing of the multipath delays is achieved which effectively eliminates intersymbol interference. Thus the receiver output behaves as though there were

a single propagation path of high strength, rather than a series of weaker paths spread in time. It should be possible to exploit this feature to send information in bauds whose durations are equal to, or even shorter than, the multipath duration.

The procedure of detecting, weighting, and combining signals with different delays clearly has an analog in the frequency domain. Using again a wide-band signal, one can build a receiver that divides the spectrum up into segments (which need be no narrower than the fine structure of the selective fading, that is, the reciprocal of the multipath duration), then detects, weights, and adds their contributions.²²

Although it is clear that the selective fading and intersymbol interference resulting when multipath is present in conventional systems can be effectively disposed of, a new and undesirable effect appears in the Rake system—a form of noise in the output. This is most easily seen when we consider the “matched-filter” interpretation of the Rake receiver.¹⁴ From this aspect, the channel multipath is cascaded with a time-reversed duplicate of itself at the receiver. This combination yields a new multipath characteristic of twice the original spread, but one in which there is now a strong central path formed of the time-reconstituted contributions of the original channel paths. Correlation detection is then employed to isolate this path and suppress the others on either side of it, thus effectively eliminating the spread. While there are thus no coherent contributions from any paths but the strong central one, the remainder of the “new” paths do produce fluctuation in the integrating filter outputs, by virtue of the finite correlation time allowed (the baud length T). This fluctuation is virtually identical to that produced by channel noise.

It can be shown²³ that the snr at the integrating filter output is proportional to TW , the product of the baud length by the reference signal bandwidth. Here the noise²⁴ is the sum of that due to interference in the channel, and that due to the correlation fluctuation just mentioned. Experiments with the prototype system in which W was fixed at 10 kc showed that even with no channel noise present, satisfactory error rates were attained only for TW about 50 or larger.

We may interpret this number as follows. For $W=10$ kc, bauds as short as 5 msec can be used. This figure should be substantially independent of the duration of

²² Such a scheme was conceived independently by Ehrick and Steinberg [45].

²³ By properly interpreting the correlator snr result [(13) of [17]], an equation can be derived for Rake output snr. It consists of the product of the dimensionless quantity TW with a time-varying quantity depending only on the multipath, the spectral shape of the reference, signal and noise, and the ratio of powers of incoming signal and noise.

²⁴ Making the baud length equal to or less than the multipath duration is a violation of the assumption under which one can attribute equal output snr to the two forms of Rake system: the delayed-signal, and delayed-reference forms (Section III-A). The delayed-reference scheme has the smaller output noise because various portions of this noise add now with delays greater than the pertinent fluctuation period (the baud duration).

the multipath provided the Rake delay line has a length at least equal to this duration, because of the delay reconstitution. Because the length of the delay line of the prototype system was only 3-msec, the 5-msec limitation on baud length made it impossible to demonstrate the full possibilities of time reconstitution. On the other hand if a bandwidth of 50 kc could have been employed, then 1-msec baud lengths should have been usable over normal ionospheric multipath.

It is recognized that at present the Rake system, like any high-order frequency-diversity scheme, is relatively uneconomical of bandwidth. On the other hand, the system performs with high reliability, and in fact may work best, over a channel having extensive multipath. Thus Rake systems may prove useful in providing satisfactory communication at those frequencies, well below the “optimum working frequency,” which would normally be abandoned on account of multipath. There is also promise that the Rake technique can be extended to the multiplexing of a considerable number of independent, wide-band transmissions into the same frequency band [49].

Another of the important directions for future work on systems of this type is the reduction of the minimum TW product required for a given error rate. Several approaches look promising. Among them are: 1) the search for reference signals with reduced short-time correlation fluctuations. 2) Schemes of cancelling out these fluctuations. The fluctuation waveform depends only on the reference waveforms (known exactly), the multipath characteristic (known almost exactly) and the sequence of Marks and Spaces transmitted (this can be deduced at the receiver with low probability of error by using a step-by-step hypothesis test). So far neither of these approaches has produced workable results.

Much improvement is probably possible in equipment size and complexity. An ingenious series of ideas has been proposed by Sunstein, Steinberg, and Ehrick [44] for time-sharing a single tap unit rather than building a multiplicity of them. Also, it would be desirable to make some present manual operations automatic, such as the complete suppression of those tap circuits not responding to any significant path.

The assumption has been made throughout that the fluctuation period of the multipath is long compared to the symbol lengths, although this is not demanded in the original development of Section III-B. In cases where the fluctuation period becomes comparable to or less than the symbol length, the Rake receiver takes a particularly simple form: instead of weighting each correlator output by a long-term measurement of itself, one need only pass it through a filter identical to the measurement filter of the present tap circuits and square the filter outputs [45, 46].

One final observation should be made: the Rake system as presently conceived does not appear to be directly adaptable to the transmission of analog, rather than digital, information. Further study may extend the

applicability of the Rake technique in this and other directions.

ACKNOWLEDGMENT

The development of the Rake system has been the result of the stimulation, support, and hard work of a number of people. We would particularly like to note the contributions of R. S. Berg, C. W. Bergman, J. W. Craig, R. E. Gay, W. C. McLaughlin, W. B. Smith, and C. A. Wagner to the system development and testing, and the assistance of the U.S. Army Signal Engineering Laboratories (Fort Monmouth, N. J.) and the Sixth Army (San Francisco) in providing field test facilities. We would like to record our great indebtedness to W. B. Davenport, Jr., whose guidance in earlier work underlying the Rake system made this concept possible. This paper reflects in many ways the influence of R. M. Fano who has spent many hours and not a few graduate students (including the authors) in pursuit of solutions to the multipath problem.

BIBLIOGRAPHY

- [1] Potter, R. K. "Transmission Characteristics of a Short-Wave Telephone Circuit," *PROCEEDINGS OF THE IRE*, Vol. 18 (April, 1930), pp. 581-648.
- [2] Eckersley, T. L. "Multiple Signals in Short-Wave Transmission," *PROCEEDINGS OF THE IRE*, Vol. 18 (January, 1930), pp. 106-122.
- [3] Crosby, M. G. "Exalted-Carrier Amplitude and Phase-Modulation Reception," *PROCEEDINGS OF THE IRE*, Vol. 33 (September, 1945), pp. 581-591.
- [4] Costas, J. P. "Synchronous Communications," *PROCEEDINGS OF THE IRE*, Vol. 44 (December, 1956), pp. 1713-1718.
- [5] Van Wambeek, S. H., and Ross, A. H. "Performance of Diversity Receiving Systems," *PROCEEDINGS OF THE IRE*, Vol. 39 (March, 1951), pp. 256-264.
- [6] DiToro, M. J. "Distant Radio-Communication Theory," paper presented at the IRE-URSI spring meeting (April 21-24, 1952), Washington, D. C. *PROCEEDINGS OF THE IRE*, Vol. 40 (June, 1952), p. 746.
- [7] Mack, A., and Levine, R. H. "A New Multichannel Teletype Terminal for Use on Long-Range High-Frequency Radio Systems," *IRE TRANSACTIONS*, Vol. CS-2 (November, 1954), pp. 161-165.
- [8] Doelz, M. L., Heald, E. T., and Martin, D. L. "Binary Data Transmission Techniques for Linear Systems," *PROCEEDINGS OF THE IRE*, Vol. 45 (May, 1957), pp. 656-661.
- [9] Bray, W. J., Lillicrap, H. G., and Owen, F. C. "The Fading Machine, and Its Use for the Investigation of the Effects of Frequency-Selective Fading," *Journal of the Institution of Electrical Engineers*, Part IIIA (London), Vol. 94, No. 12 (1947), pp. 283-297.
- [10] Granlund, J., and Stutt, C. A. See Appendix II of [11].
- [11] Arguimbau, L. B., Granlund, J., Manna, E. E., and Stutt, C. A. *Transatlantic Frequency-Modulation Experiments*, Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Mass., Technical Report 278; September 20, 1954.
- [12] Arguimbau, L. B. and Grandlund, J. "Sky-Wave FM Receiver," *Electronics*, Vol. 22 (December, 1949), pp. 101-103.
- [13] Lindsay, D. G. "A Dual-Diversity Frequency-Shift Receiver," *PROCEEDINGS OF THE IRE*, Vol. 39 (June, 1951), pp. 598-612.
- [14] Price, R. *Statistical Theory Applied to Communication Through Multipath Disturbances*, Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Mass., Technical Report 266, pp. 8-10; September, 1953.
- [15] Fano, R. M. "On Matched-Filter Detection in the Presence of Multipath Propagation," Informal Memorandum, Research Laboratory of Electronics, Massachusetts Institute of Technology; 1956.
- [16] Fano, R. M. *Signal-to-Noise Ratio in Correlation Detectors*, Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Mass., Technical Report 186, p. 11; February 19, 1951.
- [17] Green, P. E., Jr. "The Output Signal-to-Noise Ratio of Correction Detectors," *TRANSACTIONS OF THE IRE*, Vol. IT-3 (March, 1957), pp. 10-18.
- [18] Helstrom, C. W. "The Resolution of Signals in White, Gaussian Noise," *PROCEEDINGS OF THE IRE*, Vol. 43 (September, 1955), pp. 1111-1118.
- [19] Brennan, D. G. "On the Maximum Signal-to-Noise Ratio Realizable from Several Noisy Signals," *PROCEEDINGS OF THE IRE*, Vol. 43 (October, 1955), p. 1530.
- [20] Wiesner, J. B., and Lee, Y. W. "Experimental Determination of System Functions by the Method of Correlation," paper presented at IRE National Convention, New York, N. Y., March, 1950. *PROCEEDINGS OF THE IRE*, Vol. 38 (February, 1950), p. 205.
- [21] Lee, Y. W. *Application of Statistical Methods to Communication Problems*, Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Mass., Technical Report 181, pp. 25-29; September 1, 1950.
- [22] Zierler, N. *Several Binary-Sequence Generators*, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Mass., Technical Report 95; September 12, 1955.
- [23] Golomb, S. W. *Sequences with Randomness Properties*, Glenn L. Martin Company, Baltimore, Maryland, Terminal Progress Report; June 14, 1955.
- [24] Huffman, D. A. "The Synthesis of Linear Sequential Coding Networks," *Proceedings of the Third London Symposium on Information Theory*, September, 1955, New York: Academic Press, pp. 77-95; 1956.
- [25] Price, R. "A Note on the Envelope and Phase-Modulated Components of Narrow-Band Gaussian Noise," *IRE TRANSACTIONS*, Vol. IT-1 (September, 1955), pp. 9-13.
- [26] Doelz, M. L., and Heald, E. T. "Predicted Wave Radio Teletype System," *IRE CONVENTION RECORD*, Part VIII (March, 1954), pp. 63-69.
- [27] Middleton, D., and Van Meter, D. "Detection and Extraction of Signals in Noise from the Point of View of Statistical Decision Theory," *Journal of the Society of Industrial and Applied Mathematics*, Vol. 3 (December, 1955), pp. 192-253, and Vol. 4 (June, 1956), pp. 86-119.
- [28] Peterson, W. W., Birdsall, T. G., and Fox, W. C. "The Theory of Signal Detectability," *IRE TRANSACTIONS*, Vol. IT-4 (September, 1954), pp. 171-212.
- [29] Lawson, J. L., and Uhlenbeck, G. E. *Threshold Signals*, New York: McGraw-Hill Book Company, Inc., section 7.5, 1950.
- [30] Woodward, P. M., and Davies, I. L. "Information Theory and Inverse Probability in Telecommunications," *Proceedings of the Institution of Electrical Engineers*, Part III, Vol. 99 (March, 1952), pp. 37-44.
- [31] Kotel'nikov, V. A. "Problems of Interference Immunity in Radio Communication," *Radiotekhnika Sbornik* (1947), pp. 5-12.
- [32] Root, W. L., and Pitcher, T. S. "Some Remarks on Statistical Detection," *IRE TRANSACTIONS*, Vol. IT-1 (December, 1955), pp. 33-38.
- [33] Woodward, P. M. *Probability and Information Theory, with Applications to Radar*, New York: McGraw-Hill Book Company, Inc., pp. 31-35, 1955.
- [34] Turin, G. L. "On the Estimation in the Presence of Noise of the Impulse Response of a Random, Linear Filter," *IRE TRANSACTIONS*, Vol. IT-3 (March, 1957), pp. 5-10.
- [35] [21], (70).
- [36] [29], Section 8.6.
- [37] Turin, G. L. *Communication Through Noisy, Random-Multipath Channels*, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Mass., Technical Report 116; May, 1956.
- [38] Turin, G. L. "Communication Through Noisy, Random-Multipath Channels," *IRE CONVENTION RECORD*, Vol. 4, Part IV (1956), pp. 154-166.
- [39] Booker, H. G., and Gordon, W. E. "A Theory of Radio Scattering in the Troposphere," *PROCEEDINGS OF THE IRE*, Vol. 38 (April, 1950), pp. 401-412.
- [40] Booker, H. G., Ratcliffe, J. A., and Shinn, D. H. "Diffraction From an Irregular Screen with Application to Ionosphere Problems," *Philosophic Transactions of the Royal Society* (London), Part A, Vol. 242 (September 12, 1950), pp. 579-607.
- [41] McNicol, R. W. E. "The Fading of Radio Waves of Medium and High Frequencies," *Proceedings of the Institution of Electrical Engineers*, Part III, Vol. 96 (November, 1949), pp. 517-524.
- [42] Price, R. "Optimum Detection of Random Signals in Noise, with Application to Scatter-Multipath Communication, I," *IRE TRANSACTIONS*, Vol. IT-2 (December, 1956), pp. 125-135.
- [43] Price, R., and Green, P. E., Jr. "Measurement of Ionospheric Path-Phase for Oblique Incidence," *Nature*, Vol. 179 (February 16, 1957), pp. 372-373.
- [44] Sunstein, D. E., Steinberg, B. D., and Ehrick, W. G. *Wholesale Rake* (General Atronic Corporation under Subcontract P.O.-

- DDL-A-4448 to Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Mass.; February–August, 1957.
- [45] Price, R. *Error Probabilities for the Ideal Detection of Signals Perturbed by Scatter and Noise*, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Mass., Group Report 34-40, section 6b, October 3, 1955. (Not generally available.)
- [46] Middleton, D. "On the Detection of Stochastic Signals in Normal Noise, Part 1," IRE TRANSACTIONS, Vol. IT-3 (June, 1957), pp. 86–121.
- [47] Smith, R. A. "The Relative Advantages of Coherent and Incoherent Detectors: A Study of Their Output Noise Spectra under Various Conditions," *Proceedings of the Institution of Electrical Engineers*, Part III, Vol. 98 (September, 1951), pp. 401–406.
- [48] Dwight, H. B. *Tables of Integrals and Other Mathematical Data*, New York: Macmillan and Company, formula 591, 1947.
- [49] Pankowski, B. J. Private communication.
- [50] Friis, H. T., and Feldman, C. B. "A Multiple Unit Steerable Antenna for Short-Wave Reception," PROCEEDINGS OF THE IRE, Vol. 25 (July, 1937), pp. 841–917.
- [51] Zierler, N. "Linear Recursive Sequences," submitted to *Journal of the Society for Industrial and Applied Mathematics*, November, 1957.
- [52] Earp, C. W. note in *Proceedings of the Institution of Electrical Engineers*, Part III, Vol. 98 (July, 1951), p. 264.
- [53] Kharkevich, A. A. "Signal Transmission by Modulated Noise," *Electrosviaz'*, Vol. 11 (1957), pp. 42–46.

Low Noise Tunable Preamplifiers for Microwave Receivers*

M. R. CURRIE†, SENIOR MEMBER, IRE, AND D. C. FORSTER†, MEMBER, IRE

Summary—An investigation of noise reduction in backward-wave amplifiers has yielded two significant results. 1) This type of amplifier has been demonstrated to be capable of very-low noise figures and therefore constitutes an entirely new class of microwave receiver tubes. Tube noise figures less than 6 db for a 25 per cent tuning range and less than 4.5 db for a 10 per cent tuning range of the amplifier's narrow pass band have been attained. 2) Tube noise figures of well under 4 db have been measured, which are the lowest recorded to date on any type of microwave tube. This performance results from a special low-noise gun which was developed for use with hollow electron beams and which features a new type of beam launching mechanism.

The experimental S-band tubes and "Christmas-tree gun" are described. Detailed noise performance of the tubes is presented, as well as other data relating to the operation of the tube as a receiver component. On the basis of these experiments, it is concluded that still lower noise figures are possible using the basic concepts of this new gun, not only for backward-wave amplifiers, but also for other types of microwave tubes.

INTRODUCTION

A fundamental requirement in virtually all microwave systems is maximum receiver sensitivity. This need has not only been intensified with the advent of new types of broad-band systems but is now also coupled with the demand for receivers possessing greater flexibility. Modern receivers must often be capable of operation over appreciable portions of an octave, sometimes with almost instantaneous tuning—and this must be accomplished while maintaining very-low noise figures

In view of these considerations, an investigation of noise reduction in backward-wave amplifiers was undertaken in an effort to adapt the unique voltage-tuned

filter characteristics of this type of tube^{1,2} to receiver applications. It was evident that if the noise figure of this device could be lowered to values approaching the ultimate theoretically predicted limit²⁻⁴ (6–7 db) it would constitute a new type of microwave receiver tube whose characteristics would potentially make possible the evolution of a special class of receivers featuring both high sensitivity and great versatility.

Some other more fundamental considerations initially motivated this investigation. Previous experiments on the nature of noise in electron beams have all utilized small cylindrical beams.^{5,6} Although the generalized one-dimensional theory predicts identical minimum noise figures for all beam-type amplifiers,⁴ extensive experiments on noise reduction have been concerned almost exclusively with the conventional traveling-wave tube.^{7,8} In addition to the application of present concepts of beam noise to a different type of tube, the use of the backward-wave amplifier permits the investigation of a range of parameters and beam geometry not

¹ R. Kompfner and N. T. Williams, "Backward-wave tubes," *Proc. IRE*, vol. 41, pp. 1602–1611; November, 1953.

² M. R. Currie and J. R. Whinnery, "The cascade backward-wave amplifier—a high gain voltage-tuned filter for microwaves," *Proc. IRE*, vol. 43, pp. 1617–1632; November, 1955.

³ T. E. Everhart, "Concerning the noise figure of a backward-wave amplifier," *Proc. IRE*, vol. 43, pp. 444–449; April, 1955.

⁴ H. A. Haus and F. N. H. Robinson, "Minimum noise figure of microwave amplifiers," *Proc. IRE*, vol. 43, pp. 981–991; August, 1955.

⁵ C. C. Cutler and C. F. Quate, "Experimental verification of space charge and transit time reduction of noise in electron beams," *Phys. Rev.*, vol. 80, pp. 875–878; December, 1950.

⁶ L. D. Smullin and C. Fried, "Microwave noise measurements on electron beams," *IRE TRANS*, vol. ED-1, pp. 168–183; December, 1954.

⁷ D. A. Watkins, "Noise reduction in beam-type amplifiers," *Proc. IRE*, vol. 40, pp. 65–70; January, 1952.

⁸ R. W. Peter, "Low-noise traveling-wave amplifier," *RCA Rev.*, vol. 13, pp. 344–368; September, 1952.

* Original manuscript received by the IRE, October 28, 1957; revised manuscript received, December 24, 1957.

† Hughes Aircraft Co., Culver City, Calif.